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3. Solving simultaneous linear equations in 2 unknowns

Solving simultaneous equations is to find the common solution of the given equations.

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4. Method of substitution Example:

 $\begin{cases} 3x + y = 7 \dots(1) \\ 2x - 3y = 1 \dots(2) \end{cases}$ From (1), $y = 7 - 3x \dots(3)$ Put (3) into (2), 2x - 3(7 - 3x) = 1, x = 2Put x = 2 into (3), y = 7 - 3(2) = 1**Ans.** x = 2, y = 1

5. Method of elimination Example:

 $\begin{cases} 4x + 3y = -9.....(1) \\ 3x + 2y = -8....(2) \\ (1) \times 2, \quad 8x + 6y = -18....(3) \\ (2) \times 3, \quad 9x + 6y = -24....(4) \\ (3) - (4), \quad 8x - 9x = -18 - (-24) \\ -x = 6, \quad x = -6 \\ Put \ x = -6 \ into \ (2), \quad 3(-6) + 2y = -8, \quad y = 5 \\ Ans. \ x = -6, \quad y = 5 \end{cases}$

6. Graphical method

Draw the graphs of the linear equations, then find their intersection point. The intersection point is the common solution of the simultaneous equations.

7. Practical problems

(a) In solving practical problems, choose two letters to represent the unknowns and state it explicitly. Remember to indicate the unit (單位), if any.

e.g. Let x be the number of tables, y be the cost of a table. Then form 2 equations according to the information of the question, and solve the simultaneous equations.

Remember to find both the values of *x* and *y*, and check whether any of the answers should be rejected (丟棄). Write down the answers clearly, including the unit (單位), if any.

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(b)

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- (d) In problems about ages, if the present ages of two persons are x and y, their ages 3 years ago would be (x-3) and (y-3) respectively.
- (e) In problems about speeds, make use of a table to organize the information. For example:

Distance (D)	Speed (S)	Time (T)	
100 km	<i>x</i> km/h	$\frac{100}{x}$ h	
100 km	y km/h	$\frac{100}{y}$ h	

Remember:

$$S=\frac{D}{T}\;;\;\;T=\frac{D}{S}\;;\;\;D=S\times T$$

8. More about solving simultaneous linear equations

- (a) Simplify each of the given equations before solving the simultaneous equations.
- (b) In equations involving $\frac{1}{x}$ and $\frac{1}{y}$, we may let them as m

and *n* respectively and then solve for *m* and *n* first. For example: (12, 2)

$$\begin{cases} \frac{12}{x} - \frac{2}{y} = 15 \dots (1) \\ \frac{16}{x} - \frac{5}{3y} = 14 \dots (2) \\ \text{Let } \frac{1}{x} = m, \frac{1}{y} = n \\ \therefore \begin{cases} 12m - 2n = 15 \dots (3) \\ 16m - \frac{5}{3}n = 14 \dots (4) \\ (3) \times 5, \quad 60m - 10n = 75 \dots (5) \\ (4) \times 6, \quad 96m - 10n = 84 \dots (6) \\ (6) - (5), \quad 96m - 60m = 84 - 75, \quad m = \frac{9}{36} = \frac{1}{4} \dots (7) \end{cases}$$

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Put (7) into (3),
$$\therefore 12(\frac{1}{4}) - 2n = 15$$
, $n = -6$
 $m = \frac{1}{4}$, $x = \frac{1}{m} = 4$; $n = -6$, $y = \frac{1}{n} = -\frac{1}{6}$
Ans. $x = 4$, $y = -\frac{1}{6}$

9. Number of solutions

- (a) Usually simultaneous linear equations in two unknowns have one set of solution, i.e. one intersection point on the graphs.
- (b) When the graphs of two linear equations are parallel lines, they have no intersection point; in other words, the simultaneous equations have no solution. In general, for the simultaneous equations:

$$\begin{cases} A_1 x + B_1 y = C_1 \\ A_2 x + B_2 y = C_2 \end{cases}$$

if $\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}$, then the simultaneous equations have

no solution. For example, the simultaneous equations x + 3y = 5 and 4x + 12y = 10 have no solution.

(c) For the simultaneous equations:

if $A_2 = \frac{B_1}{B_2} = \frac{C_1}{C_2}$, then the two equations are in fact the

same, and their graphs overlap. In this case, the simultaneous equations have infinitely (無限的) many solutions. For example, the simultaneous equations x + 3y = 5 and 4x + 12y = 20 have infinitely many solutions.

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1. Complete the following table and draw the graph of:



- 2. (a) The point (5, -7) lies on the graph of 3x + ky 1 = 0. Find the value of k.
- (b) The point $(h, \frac{5}{2})$ also lies on the graph. Find the value of h.
- 3. (a) Complete the table and draw the graph of:

2x+y-1=0.	x	-2	0	3	4
\sim	У				

(b) Complete the table and draw the graph of:

3x - 2y = 12.	x	0	4	6	8
	У				

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(c) Solve the following simultaneous equations graphically:



4. Solve the following simultaneous equations by the method of substitution.

(a)
$$\begin{cases} y = 8x - 19 \\ y = 6x - 15 \end{cases}$$
 (b)
$$\begin{cases} x = 2y + 13 \\ 3x + y = 4 \end{cases}$$

(c)
$$\begin{cases} 3y - 7x = 1 \\ y + 2x = 9 \end{cases}$$
 (d)
$$\begin{cases} 2y = 5x + 5 \\ 2y = 6x + 8 \end{cases}$$

(e)
$$\begin{cases} 8x + 8 = 3y \\ 2y = 11 - 6x \end{cases}$$
 (f)
$$\begin{cases} 5x + 3y = 7 \\ 5y - 3x = 6 \end{cases}$$

5. Solve the following simultaneous equations by the method of elimination.

(a)
$$\begin{cases} x + y = 11 \\ x - y = 7 \end{cases}$$
 (b) $\begin{cases} x + 2y = 3 \\ x - y = 9 \end{cases}$

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 $\begin{cases} 4x - y = 18 \\ x + 2y = -9 \end{cases}$ (d) $\begin{cases} 5x + 3y = 2 \\ 3x + 9y = -6 \end{cases}$ (c)

(e)
$$\begin{cases} 3y + 2x = 2 \\ 2y - 3x = -16 \end{cases}$$
 (f) $\begin{cases} 5x + 5y = 1 \\ 3x - 2y = 5 \end{cases}$

- Solve the following simultaneous equations. 6.
- 3x + y 120 = 3x 2y = 10 + 2x(a)
- (b) 9x + 3y + 5 = x 2y + 7 = 7x 3y 4
- (c) $\frac{5x}{3} y = \frac{x}{3} + y = 6$
- (d) $\frac{6x-3y}{4} = \frac{12x+5y}{30} = 6$
- 7. Solve the following simultaneous equations.
- (a) $\begin{cases} 9x 5y 11 = 0\\ 3x 5y + 13 = 0 \end{cases}$ (c) $\begin{cases} 10x + y = 3(x + y)\\ 9(x y) = x y 32 \end{cases}$ $\begin{cases} 0.35x - 0.15y = 1\\ 0.2x + 0.1y = 1.5 \end{cases}$ (b)
- $\begin{cases} 3(x-y) = -(7+2y) \\ 3(2x-y) = 2+y \end{cases}$ (d)

(e)
$$\begin{cases} 3x + 2y = 4(4 + x) + 1\\ 5x + 3y = 7(4 + x) - 2 \end{cases}$$

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(f)
$$\begin{cases} -2(2+4y) = 5x - 5y \\ 3(1-x) + 2(y+1) = y - x \end{cases}$$

- (g) $\begin{cases} 4(x-y) + 2(x+y) = -(19+y) \\ 5(x+y) (2x+y) = -5 \end{cases}$
- 8. Solve the following simultaneous equations.

(a)
$$\begin{cases} x = 2y + 8 \\ \frac{x}{3} + 4 = y \end{cases}$$
 (b)
$$y = \frac{3}{2}x - 4 \\ x = -\frac{2}{3}y$$

(c)
$$\begin{cases} \frac{2y+x}{3} = 3\\ y - \frac{2}{3}x = 1 \end{cases}$$
 (d)
$$\begin{cases} \frac{x}{5} - \frac{3y}{2} = 3\\ \frac{3x}{8} - \frac{5y}{2} = 5 \end{cases}$$
 (e)
$$\begin{cases} \frac{5x}{3} - y = -17\\ \frac{x}{3} - \frac{y}{2} = 4 \end{cases}$$
 (f)
$$\begin{cases} \frac{x}{3} - y = -5\\ \frac{3y}{2} + \frac{2x}{3} = \frac{26}{3} \end{cases}$$

9. Solve the following simultaneous equations.



10. Find two numbers whose sum is 91 and whose difference is 17.

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- 11. One-fourth of the sum of two numbers is 16, and twice of their difference is 36. Find the numbers.
- The difference of two numbers is three-fifths of their sum. 12. and the bigger number exceeds three times the smaller number by 14. Find the numbers.
- If 9 is added to the numerator (分子) and the denominator 13. (分母) is multiplied by 2, a fraction reduces to $\frac{2}{3}$. If one is subtracted from the numerator, the fraction reduces to
 - $\frac{1}{2}$. Find the fraction.
- 3 tables and 4 chairs cost \$2800 while 14. 4 tables and 9 chairs cost \$4650. Find the prices of a chair and a table.



- 2m-3n+10, 4m-5n+6, and m+n-9 are the lengths of 15. the sides of an equilateral triangle. Find its perimeter.
- The figure shows a rectangle 16. ABCD. x+3 y+5Find the values of *x* and *y*. (a) Find the perimeter of ABCD. (b) Find the area of ABCD. (c)

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- 17. A box of pencils is shared among a group of students. If each student gets 6 pencils, there will be 5 pencils left. If each one gets 7 pencils, 8 more pencils will be needed. Find the number of students and the number of pencils in the box.
- 18. There are some \$2 coins and \$5 coins in a money box. The number of coins is 30 and their total value is 90 dollars. Find the total number of coins in the money box.
- 19. A two-digit number is equal to 8 times the sum of its digits. The number formed by reversing (顛倒位置) its digits is greater than one-third of the original number by 3. Find the number.
- 20. Find the values of A and B in the following identity: $x^2 + Ax - A \equiv (x + B)(x + 3) - 11$
- 21. David is 13 years older than Bobby. 11 years ago, Bobby's age was two-thirds of David's age. Find their present ages.

22. The ages of A and B are in the ratio 5:3. Nine years later, the ratio of their ages will be 4:3. Find their present ages.

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- 23. Two cars started at the same time to meet each other from two towns 79 km apart on a straight road. The speed of one car is 12 km/hr faster than that of the other and they met after 30 minutes. Find their speeds.
- 24. Two cars 280 km apart setting out at the same time will meet in 4 hours if they travel in opposite directions. But they will be together in 14 hours if they drive in the same direction. Find their speeds.
- 25. A boat sailing downstream covered a distance of 108 km in 6 hours. In the return trip, it took 9 hours to get back to the starting point. Find the speed of the boat in still water (靜止的水) and the speed of the water current.



 A ship sailed 360 km in 22.5 hours when sailing upstream. However, it took 2.5 hours less when sailing the same journey downstream, that is, along the current (順流). Find the speed of the ship against the current (逆流).

(II) Stimulating items, No.27-48



27. Solve the following simultaneous equations:

$$\begin{cases} \frac{4x+3y}{10} - \frac{2x-y}{5} = \frac{x-y}{2} \\ 8y - \frac{5x-2}{3} = 2x+y \end{cases}$$

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/5x+y

=0

x+b)

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- 33. (a) Under what condition will two straight lines never intersect?
- (b) If the following simultaneous equations have no solution, find the value of r: $\begin{cases} 6x - 3y + 2 = 0\\ 4x + ry + 4 = 0 \end{cases}$
- 34. The figure shows that the graphs of four given linear equations intersect at P.
- (a) Find the coordinates of P.
- (b) Find the values of *a* and *b*.

35. If a + b = 2m, a - b = 2n, find the value of $\frac{a}{b}$ in terms of *m* and *n*.

36. Given that $\begin{aligned} x_1 + x_2 &= 10 \\ x_2 + x_3 &= -3 \\ x_3 + x_1 &= 9 \\ x_1 + x_2 + x_3 &= -3 \end{aligned}$

- (a) Find the value of y.
- (b) Hence, find the values of x_1 , x_2 and x_3 .

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- 37. The cost \$C for printing a certain magazine is given by C = a + kn where *a*, *k* are constants and *n* is the number of magazines printed. When n = 1000, C = 16500; when n = 5000, C = 34500.
- (a) Find the constants *a* and *k*.
- (b) What is the average printing cost of one magazine when 40000 copies are printed?
- 38. A has \$x and B has \$y. If A gives B \$240, B will have 2 times as much as A. If B gives A \$1,010, A will have 3 times more than B. How much do they have altogether?
- 39. There are some \$2 coins in Bag P, and some \$5 coins in Bag Q. If 22 coins are taken out from Bag P and put into Bag Q, the number of coins in Bag Q will be twice that of Bag P, and the amount in Bag Q will be \$46 more than that of Bag P. Find the original numbers of coins in the two bags.



40. 4 years ago Adam was 4 times as old as his sister, and in 8 years' time he will be two times as old as her. What is the sum of their present ages?

A woman has two daughters whose difference in ages is 4 years. The sum of the ages of the woman and her daughters is 50 years old. When the elder daughter reaches her mother's present age, the mother will be 63 years old. Find their present ages.

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- 42. *A*, *B* and *C* ran in the same direction from a starting point. *A* started first at 10:00 a.m., *B* started at 10:02 a.m., and *C* started at 10:06 a.m. At 10:12 a.m. *B* overtook *A*, while *C* overtook A at 10:18 a.m. If the speed of *C* is faster than that of *B* by 1.5 m/s, find the speeds of *A*, *B* and *C* in m/s.
- 43. A boat travelled against the current from A to B and immediately returned from B to A. It saved 2 hours in the return trip, and the distance between A and B is 315 km. Besides, the time taken to travel 28 km against the current is the same as that of travelling 36 km along the current. If the speed of a boat in still water is *x* km/h, and the speed of the current is *y* km/h, find the values of *x* and *y*.
- 44. Two pipes *A* and *B* are used to fill an empty pool. If both pipes are used, it takes 3 hours to fill up the pool. However, if only pipe *A* is used for the first four hours, the rest will be filled up in 2 hours by using both pipes. Find the time taken by each pipe to fill up the pool alone.



- 45. *A* takes twice the time to complete a project as *B* does. If they work together, the project can be completed in 14 days. How long will it take each of them to complete the project alone?
- 46. In doing a project, a group of workers can complete the work in a certain number of days. If there are 8 more workers, the job will be completed 10 days sooner. If there are 8 workers fewer, the job will be delayed by 20 days. Find the numbers of workers and days required originally.

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- 47. The cost of 1 apple, 3 oranges and 2 pears is \$22.6, while the cost of 5 apples, 8 oranges and 3 pears is \$62.6.
- (a) Prove that the total cost of 1 orange and 1 pear is \$7.2.
- (b) The cost of 5 oranges is equal to that of 4 pears. Find the cost of each type of fruit.



48. A youth centre has 181 male members and 154 female members participating in two projects. Each member only joins one of the projects. In project A, the number of males exceeds twice of the number of females by 3, while in project B the number of females is three times that of the males. How many male and female members are there in each project?

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