Unit 1: Directed numbers

⚠ Important facts ⚠

1. Key terms
- ♦ directed numbers (向數), positive numbers (正數), zero (零), negative numbers (負數)
- ♦ positive sign (正號), negative sign (負號)
- ♦ ascending order (由小至大), descending order (由大至小)
- ♦ integer (整數)

2. Addition and subtraction (加法和減法):
$$+ (+b) = + b, \ + (-b) = -b, \ - (+b) = -b, \ - (-b) = +b$$

3. Multiplication (乘法):
$$(+a) \times (+b) = +ab, \ (+a) \times (-b) = -ab,$$
$$(-a) \times (+b) = -ab, \ (-a) \times (-b) = +ab$$

4. Division (除法):
$$\frac{(+a)}{(+b)} = +\frac{a}{b}, \ \frac{(+a)}{(-b)} = -\frac{a}{b}, \ \frac{(-a)}{(+b)} = -\frac{a}{b}, \ \frac{(-a)}{(-b)} = +\frac{a}{b}$$

5. Removing brackets
$$a + (b + c) = a + b + c, \ a + (b - c) = a + b - c,$$
$$a - (b + c) = a - b - c, \ a - (b - c) = a - b + c$$
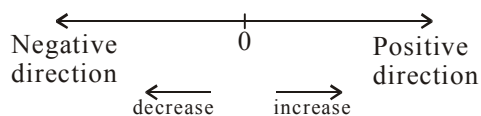
6.
In mixed arithmetic operations, remove brackets first. Then do multiplication and division before addition and subtraction.

7. Powers (次方)
$$(-1)^1 = -1, \ (-1)^2 = +1, \ (-1)^3 = -1, \ (-1)^4 = +1$$
$$(-3)^1 = -3, \ (-3)^2 = +(3^2), \ (-3)^3 = -(3^3), \ (-3)^4 = +(3^4)$$
$$(-1)^{2n} = +1, \ (-1)^{2n+1} = -1$$

8. Fractions (分數)

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}; \quad -\frac{1}{3} + \frac{1}{2} = \frac{-2}{6} + \frac{3}{6} = \frac{-2+3}{6} = \frac{1}{6}$$

9. Ordering numbers on a number line

Negative direction   0   Positive direction

decrease   increase

e.g. $5 > 3$ but $-5 < -3$; $\frac{6}{7} > \frac{2}{7}$ but $-\frac{6}{7} < -\frac{2}{7}$

10. Comparing fractions

e.g. Compare $-\frac{13}{18}$ and $-\frac{11}{15}$, the L.C.M. of the denominator is 90,

$$-\frac{13}{18} = -\frac{13 \times 5}{18 \times 5} = -\frac{65}{90}, \quad -\frac{11}{15} = -\frac{11 \times 6}{15 \times 6} = -\frac{66}{90}$$

$$\therefore -\frac{65}{90} > -\frac{66}{90}, \quad \therefore -\frac{13}{18} > -\frac{11}{15}$$

(I) Warm-up items, No.1-20 *(Time: ~45 min)*

1. Fill in the blanks with the greater sign (>) or the smaller sign (<).

(a) -9 _____ -8 (b) 0 _____ -3

(c) $-\frac{2}{7}$ _____ $-\frac{2}{5}$ (d) -0.078 _____ -1.023

2. Arrange each of the following sets of numbers in ascending order.

(a) $9.99, \ -0.87, \ 13, \ -5, \ -8$ (b) $-\dfrac{13}{7}, \ 1, \ 0, \ -2, \ \dfrac{8}{7}$

3. Arrange each of the following sets of numbers in descending order.

(a) $-\dfrac{4}{5}, \ \dfrac{1}{2}, \ -\dfrac{5}{6}, \ \dfrac{4}{5}, \ \dfrac{5}{6}, \ -\dfrac{1}{2}$

(b) $-\dfrac{13}{3}, \ -4, \ 3, \ -\dfrac{15}{4}, \ 2, \ -14$

4. Evaluate:

(a) $25 - 18 + 2$ (b) $-42 - 19 - 39 + 21$

(c) $(-17) - (-14) + (+17)$

(d) $(+33) + (-15) - [(-4) + (+6)]$

(e) $-11\dfrac{3}{4} + 12\dfrac{1}{3} + 5$ (f) $9.34 - 15.17 - 4.66 + 2.53$

5. Evaluate:

(a) $(+9) \times (-6) \div (-18)$ (b) $\dfrac{(-4) \times (-3)}{(-6) \times 10}$

(c) $\dfrac{2}{3} \times (-\dfrac{5}{8}) \div \left(-\dfrac{1}{6}\right)$

(d) $1.6 \times (-0.25) \div (-1.2) \times (-0.3)$

6.   Fill in the missing numbers.

(a) $(-19) + (\quad) = 7$ (b) $(\quad) - (-28) = -8$

(c) $(\quad) \div (-4) = -15$ (d) $(\quad) \times 6 = -10$

7.   Evaluate:

(a) $-36 \div [\, 3 \times (-2 + 5)\,]$

(b) $(-10 - 15 + 13) \div 4 \times (-6)$

(c) $30 \div (-5) - [44 \div (-4) + 1]$

(d) $(7-9) \times [15 + (-18) \div (-6)]$

8.   Evaluate:

(a) $-\dfrac{8}{3} \times \dfrac{5}{2} \div (-9) + 1$ (b) $(\dfrac{7}{4})(-\dfrac{2}{3}) + (-\dfrac{13}{24})(\dfrac{8}{39})$

(c) $\dfrac{(7-12) \times (-6)}{[-10 - (-8)] \times 10}$

9.   Numerical substitution

(a)   If $x = -3,\ y = -5,$ find $x^2 - 4y$.

(b)   If $a = -2,\ b = -6,$ find $ab + a^2$.

(c)   If $m = -\dfrac{1}{3},$ find $3m^2 + 2m - 4$.

(d)   If $x = -8,\ y = \dfrac{2}{5},\ z = -3,$ find $xy + yz + zx$.

10.　The sum of a number and −26 is −53. Find the number.

11.　If the temperature changes from 27° C to −7° C, what is the drop in ° C?

12.　Yesterday, the average temperature of Hong Kong was 8°C. It was 12°C higher than that of London. Find the temperature of Beijing if it was 5°C higher than that of London.

13.　A submarine at 150 m below sea level launches a rocket 250 m above sea level. How high is the rocket above the submarine?

14.　A shopkeeper gained $80 in selling an article but then lost $150 in selling another article. How much did he actually gain or lose?

15.　In a mathematics test, students scored 2 marks for a correct answer, −2 marks for a wrong answer and −1 mark for not answering the question. If Andrew got 32 questions correct, 10 questions wrong and 8 questions unanswered, what was his score in the test?

16. *p, q* are two numbers on the number line. *p* is 6 units from the left of -7, while *q* is 5 units from the right of -7. Find the values of:

(a) $p - q$ (b) $p + q$
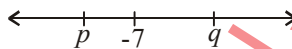
17. Find the values of the following.

(a) $(-2)^3 \times 4 + 3$ (b) $[(-8) - (-6)]^4$

(c) $4(-5)^3 \div (-15)^2$ (d) $(-6)^2 - (-2)^5$

(e) $\dfrac{4}{\left(-\dfrac{1}{3}\right)^2}$

18. Find the values of the following.

(a) $(3\frac{1}{2} - 7\frac{1}{6}) \div [-2^2 - (-\frac{2}{3})^2]$

(b) $-[5 - (-4 + 7)] \times (-6)^2 - (-2^2)$

(c) $\dfrac{(-6)^2 - (10) + (-2)^3}{-(3^2)}$

19. Find the value of n if $(3+n)^2$ is equal to zero.

20. Evaluate $(-1)^{2005} - (-1)^{2006}$.

(II) Stimulating items, No. 21-32

21. Find the values of the following.

    (a) $\quad -\left(1\dfrac{1}{2}\right)^{2} \div \left[\left(-\dfrac{1}{2}\right)^{3} + \left(\dfrac{-0.5}{\dfrac{1}{2}}\right)\right]$

    (b) $\quad \left(\dfrac{1.3}{-3.9}\right) - \left\{\left(\dfrac{-3}{4}\right) - \left[-\dfrac{1}{3} - \left(-\dfrac{1}{5} + 0.7\right)^{2}\right]\right\}$

22. Find the values of the following.

    (a) $\quad [(-3^{4}) - (-1)^{3}] \times \left[\left(-3\dfrac{1}{2}\right) \times \left(-\dfrac{5}{14}\right) + \left(-\dfrac{4}{5}\right)\right]$

    (b) $\quad \left[2\dfrac{1}{6} + \left(-2\dfrac{1}{4}\right)\right] \div \left(0 - \dfrac{1}{3}\right) - \left[15 \div \left(-6\dfrac{2}{3}\right)\right]$

23. Evaluate $1 - \dfrac{1}{\left[1 - \dfrac{1}{1 - (\dfrac{1}{-3})^{2}}\right]^{3}}$.

24. Given: $a = -5,\ b = -\dfrac{1}{4},\ c = 0.3,\ d = -2,\ e = 1$. Find the value of each of the following expressions.

    (a) $\quad \dfrac{(-d)(-e)}{-(c + ab)}$ 	(b) $\quad \dfrac{-(bc - de)}{-(a)^{2}}$

25. Solve each of the following equations.

(a) $\dfrac{x}{-25} - (\dfrac{-3}{10})^2 = -(\dfrac{-1}{2})^2$

(b) $(0.75)(\dfrac{-2}{3}) - (0.8)(-0.625x) = (0.5)(\dfrac{-2}{3})$

26. Evaluate:

(a) $(99) \times (98) \times (97) \times \ldots \times (-97) \times (-98) \times (-99)$

(b) $(-1) + (+2) + (-3) + (+4) + \ldots + (+50)$

27. Evaluate:

$(-1\dfrac{1}{2}) \times (+1\dfrac{1}{3}) \times (-1\dfrac{1}{4}) \times (+1\dfrac{1}{5}) \times \ldots \times (-1\dfrac{1}{80})$

28. Given $y = x^2 + x^3 + \ldots + x^{100}$.

(a) What is the value of y when $x = 1$?

(b) What is the value of y when $x = -1$?

29. a, b and c are three different integers, and it is known that $abc = 60$ and $a+b+c = 10$.

(a) Explain why two of these three numbers must be negative numbers.

(b) If a is the smallest number and b is the largest number, find the values of a, b and c.

30. In a test, there were 50 multiple choice questions. 4 marks were awarded for each correct answer. However, -2 marks were given for an unanswered question, and -3 marks for an incorrect answer.

(a) If a student left 5 questions unanswered, find the highest possible marks he might get.

(b) If a student answered all the questions and got 123 marks in the test, find his percentage of correct answers.

31. In each of the following, state whether the given statement is always true. If it is not always true, explain by giving an example.

(a) "$-a$" must be a negative number.

(b) $-y$ and $+y$ must be two different numbers.

(c) If the product of two numbers is negative, then the two numbers must be of opposite signs.

(d) If a is positive and b is negative, then "$a - b$" must be positive.

(e) If the sum of two numbers is positive, then both numbers must be positive.

32. The table below shows the time differences between Hong Kong and some cities:

Bangkok	London	New York	Sydney	Tokyo	Vancouver
−1 h	a h	−12 h	+2 h	+1 h	b h

A positive time difference means the time in the city is ahead that of Hong Kong, while a negative time difference means the opposite, that is, behind that of Hong Kong.

(a) Calculate the time difference between New York and Sydney, and describe the difference in words.

(b) Find the time difference between Bangkok and London in terms of a.

(c) When Sydney is 7:00 p.m. on Thursday, London is 10:00 a.m. on Thursday. Find the value of a.

(d) When Hong Kong is 10:00 a.m. on Tuesday, Vancouver is 7:00 p.m. on Monday. Find the value of b.

(e) (i) Find the time difference between Tokyo and Vancouver, and describe the difference in words.

    (ii) It takes 14 hours to fly from Tokyo to Vancouver. If a person leaves Tokyo at 9:00 am on Friday, find the Vancouver time when he arrives at there.