Answers & Explanatory notes

UNIT 1 RATIONAL AND IRRATIONAL NUMBERS

1. A	2. D	3. D	4. D	5. B	6. C	7. A	8. A
9. B	10. A	11. C	12. A	13. C	14. B	15. B	16. B
17. D	18. A	19. C	20. D	21. D	22. D	23. A	24. C
25. A	26. B	27. B	28. A	29. B	30. C	31. A	32. D
33. B	34. D	35. C	36. D	37. A	38. C	39. B	40. C
41. B	42. D	43. C	44. B	45. A	46. A	47. D	48. C
49. C	50. B	51. A	52. B	53. A	54. D	55. C	56. A
57. A	58. C	59. A					

Explanatory Notes

6.
$$\begin{array}{r} 0.35999\ldots \\ -\,0.19999\ldots \\ \hline 0.16 \end{array}$$

$\therefore\ 0.35\dot{9} - 0.1\dot{9} = 0.16 = \dfrac{4}{25}$

7. I. $\sqrt{2} + (-\sqrt{2}) = 0$ which is rational, $\therefore$ true.

II. $\sqrt{2} \times \sqrt{3} = \sqrt{6}$ which is irrational, $\therefore$ not true.

III. π^2 is irrational, $\therefore$ not true.

$\therefore$ The answer is A.

21. $\sqrt{0.0343} = \sqrt{\dfrac{343}{10000}} = \dfrac{7\sqrt{7}}{100} = \dfrac{7p}{100}$

40. $\sqrt{18}x = 1$, $3\sqrt{2}x = 1$, $x = \dfrac{1}{3\sqrt{2}}$, $\therefore\ x = \dfrac{\sqrt{2}}{6}$

42. $(\sqrt{7}+3)y = 2$, $y = \dfrac{2}{\sqrt{7}+3}$, $y = \dfrac{2(\sqrt{7}-3)}{7-9}$,

$\therefore y = -(\sqrt{7}-3) = 3 - \sqrt{7}$

43. I. $m + n = \dfrac{\sqrt{5}-6}{2} + \dfrac{\sqrt{5}+6}{2} = \dfrac{2\sqrt{5}}{2} = \sqrt{5}$ which is irrational

II. $m^2 + n^2 = (\dfrac{\sqrt{5}-6}{2})^2 + (\dfrac{\sqrt{5}+6}{2})^2$

$= \dfrac{5 - 12\sqrt{5} + 36 + 5 + 12\sqrt{5} + 36}{4} = \dfrac{41}{2}$ which is rational

III. $mn = (\frac{\sqrt{5}-6}{2})(\frac{\sqrt{5}+6}{2}) = \frac{5-36}{4} = -\frac{31}{4}$ which is rational

$\therefore$ The answer is C.

44. $\frac{\sqrt{14}}{\sqrt{2}+\sqrt{7}} = \frac{\sqrt{14}(\sqrt{2}-\sqrt{7})}{2-7} = \frac{\sqrt{28}-\sqrt{98}}{-5} = \frac{7\sqrt{2}-2\sqrt{7}}{5}$

45. $\frac{6}{3\sqrt{5}-5\sqrt{3}} = \frac{6(3\sqrt{5}+5\sqrt{3})}{45-75} = \frac{6(3\sqrt{5}+5\sqrt{3})}{-30} = -\frac{3\sqrt{5}+5\sqrt{3}}{5}$

46. $\frac{\sqrt{6}+1}{7-3\sqrt{6}} = \frac{(\sqrt{6}+1)(7+3\sqrt{6})}{49-54} = \frac{10\sqrt{6}+25}{-5} = -2\sqrt{6}-5$

47. $0.\dot{5} = \frac{5}{9}$, $0.\dot{1}\dot{5} = \frac{5}{33}$, $\therefore$ $0.\dot{5}+0.\dot{1}\dot{5} = \frac{5}{9}+\frac{5}{33} = \frac{70}{99}$

48. A. $\sqrt{2}-\sqrt{2}=0$ which is rational

B. $\sqrt{3}\times\sqrt{3}=3$ which is rational

D. $\frac{0}{\sqrt{5}}=0$ which is rational

$\therefore$ The answer is C.

51. $\sqrt{24}=n$, $2\sqrt{2}\times\sqrt{3}=n$, $2m\sqrt{2}=n$, $\therefore$ $\sqrt{2}=\frac{n}{2m}$

52. $x=\sqrt{45}=3\sqrt{5}$, $y=\sqrt{80}=4\sqrt{5}$, $\therefore$ $\sqrt{5}=\frac{x}{3}=\frac{y}{4}$, $\therefore$ $y=\frac{4x}{3}$

53. $(\sqrt{a}-\frac{1}{\sqrt{a}})^2-(\sqrt{a}+\frac{1}{\sqrt{a}})^2=(a-2+\frac{1}{a})-(a+2+\frac{1}{a})=-4$

54. $y-\frac{1}{y}=2\sqrt{6}$, $(y-\frac{1}{y})^2=(2\sqrt{6})^2$, $y^2-2+\frac{1}{y^2}=24$,

$\therefore$ $y^2+\frac{1}{y^2}=26$

55. $\frac{a-3\sqrt{a}}{a+\sqrt{a}} = \frac{(a-3\sqrt{a})(a-\sqrt{a})}{a^2-a} = \frac{a^2-4a\sqrt{a}+3a}{a(a-1)} = \frac{a-4\sqrt{a}+3}{a-1}$

56. $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{12}} = \frac{\sqrt{2}}{2}+\frac{1}{2\sqrt{3}} = \frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{6} = \frac{3\sqrt{2}+\sqrt{3}}{6} = \frac{3x+y}{6}$

57. $\frac{\sqrt{15}+4}{\sqrt{15}-4}-\frac{\sqrt{15}-4}{\sqrt{15}+4} = \frac{(\sqrt{15}+4)^2-(\sqrt{15}-4)^2}{15-16}$

$=(\sqrt{15}-4)^2-(\sqrt{15}+4)^2=(15-8\sqrt{15}+16)-(15+8\sqrt{15}+16)$

$=-16\sqrt{15}$

58. $a=\frac{1}{3-\sqrt{10}}=\frac{3+\sqrt{10}}{9-10}=-(3+\sqrt{10})$,

$b=\frac{1}{3+\sqrt{10}}=\frac{3-\sqrt{10}}{9-10}=\sqrt{10}-3$

I. $a-b=-(3+\sqrt{10})-(\sqrt{10}-3)=-2\sqrt{10}$ which is irrational

II. $a+b=-(3+\sqrt{10})+(\sqrt{10}-3)=-6$ which is rational

III. $ab=-(3+\sqrt{10})(\sqrt{10}-3)=-(10-9)=-1$ which is rational

IV. $\dfrac{a}{b}=\dfrac{-(3+\sqrt{10})}{\sqrt{10}-3}=\dfrac{-(3+\sqrt{10})^2}{10-9}=-(19+6\sqrt{10})$

which is irrational

$\therefore$ The answer is C.

59. $(\sqrt{7-3\sqrt{5}})(\sqrt{7+3\sqrt{5}})=\sqrt{(7-3\sqrt{5})(7+3\sqrt{5})}=\sqrt{49-45}=2$

UNIT 2 LAWS OF INDICES (2)

1. B	2. B	3. D	4. C	5. C	6. A	7. D	8. A
9. D	10. B	11. B	12. C	13. A	14. C	15. B	16. A
17. C	18. D	19. C	20. D	21. C	22. B	23. A	24. B
25. D	26. A	27. C	28. B	29. D	30. D	31. B	32. C
33. A	34. D	35. A	36. D	37. C	38. B	39. C	40. B
41. B	42. B	43. D	44. A	45. B	46. A	47. C	48. B
49. C	50. D	51. A	52. D	53. B	54. B	55. C	56. A
57. B	58. C	59. C	60. A	61. D	62. A	63. C	64. D
65. D	66. B	67. A	68. C	69. D	70. C	71. B	72. A
73. B	74. A	75. B	76. C	77. D	78. A	79. C	

Explanatory Notes

11. $=[-(a^{-1})^{-2}]^{-3}=(-a^2)^{-3}=-a^{-6}=-\dfrac{1}{a^6}$

12. $=\dfrac{(2^3)^{-3}}{(2^2)^6}\times\dfrac{2^7}{(2^5)^{-1}}=\dfrac{2^{-9}}{2^{12}}\times\dfrac{2^7}{2^{-5}}=2^{-9}$

15. $=6x^{-3}y^2\times(2x^{-1}y^2)=12x^{-4}y^4=\dfrac{12y^4}{x^4}$

19. $=(a^{-5}b)^2(ab^{-1})^{-4}=(a^{-10}b^2)(a^{-4}b^4)=a^{-14}b^6=\dfrac{b^6}{a^{14}}$

20. $=(\dfrac{1}{m}-\dfrac{1}{n})^{-1}=(\dfrac{n-m}{mn})^{-1}=\dfrac{mn}{n-m}$

27. $27^x=(3^3)^x=(3^x)^3=y^3$

28. $4^{x+2}=4^x\cdot 4^2=16y$

29. $=4\cdot 2^x=2^2\cdot 2^x=2^{x+2}$

32. $5^{2x+1}=5^{2x}\cdot 5=(5^x)^2\cdot 5=5y^2$

34. $=\dfrac{3^{2n-1}\cdot 3^{3n+3}}{3^{6n}}=3^{2-n}$

35. $=\dfrac{3^{n+2}\cdot 5^{n+2}}{3^{n+1}\cdot 5^{n-1}}=3\cdot 5^3$

45. $49\cdot 7^{4y-1}=(2006y)^0$, $7^2\cdot 7^{4y-1}=1$, $7^{4y+1}=7^0$, $4y+1=0$,
$\therefore\ y=-\dfrac{1}{4}$

46. $32^m\cdot 8^{m+2}=\dfrac{1}{16}$, $2^{5m}\cdot 2^{3m+6}=2^{-4}$, $8m+6=-4$, $\therefore m=-\dfrac{5}{4}$

49. $2^{n+2}-2^n=48$, $2^n(2^2-1)=48$, $2^n=16$, $\therefore\ n=4$

50. $10^{k-2}-10^{k+1}+999=0$, $10^k(10^{-2}-10)=-999$, $10^k(-\dfrac{999}{100})=-999$,
$10^k=100$, $\therefore\ k=2$

61. $a^2=2^{-1}$, $(a^2)^{-3}=(2^{-1})^{-3}$, $\therefore a^{-6}=2^3=8$

62. $4a=3b=y$, $\therefore\ a=\dfrac{y}{4}$ and $b=\dfrac{y}{3}$,
$\therefore\ a^{-2}b^3=(\dfrac{y}{4})^{-2}(\dfrac{y}{3})^3=(\dfrac{16}{y^2})(\dfrac{y^3}{27})=\dfrac{16y}{27}$

63. $=1\div(\dfrac{2}{a}+\dfrac{1}{b})=1\div\dfrac{2b+a}{ab}=\dfrac{ab}{a+2b}$

64. $=(x+y)\div(\dfrac{1}{x^2}-\dfrac{1}{y^2})=(x+y)\div\dfrac{y^2-x^2}{x^2y^2}=(x+y)\times\dfrac{x^2y^2}{(y-x)(y+x)}$
$=\dfrac{x^2y^2}{y-x}$

65. $x-\dfrac{1}{x}=3$, $(x-\dfrac{1}{x})^2=3^2$, $x^2-2+\dfrac{1}{x^2}=9$, $\therefore x^2+\dfrac{1}{x^2}=11$

68. $=4^{n-1}(3\cdot 4^2-5)=43\cdot 4^{n-1}$

69. $=3^{2n-2}+3^{2n}=3^{2n-2}(1+3^2)=10\cdot 3^{2n-2}$

70. $=\dfrac{3^n(7+6\cdot 3)}{3^n\cdot 3^{-2}}=25\cdot 3^2=225$

71. $=\dfrac{4\cdot 5^{2n-2}-6\cdot 5^{2n-1}}{5^{2n}+5^{2n}}=\dfrac{5^{2n-2}(4-6\cdot 5)}{2\cdot 5^{2n}}=\dfrac{5^{-2}(-26)}{2}=-\dfrac{13}{25}$

73. $5^k+5^{k-1}=0.24$, $5^k(1+5^{-1})=\dfrac{6}{25}$, $5^k(\dfrac{6}{5})=\dfrac{6}{25}$, $5^k=\dfrac{1}{5}$, $\therefore$
$k=-1$

74. $5\cdot 3^{y-1}+3^{y+2}-\dfrac{6^{-2}}{2^{-7}}=0$, $3^y(5\cdot 3^{-1}+3^2)=\dfrac{6^{-2}}{2^{-7}}$, $3^y(\dfrac{32}{3})=\dfrac{2^7}{6^2}$,
$3^y=\dfrac{2^7}{2^2\cdot 3^2}\times\dfrac{3}{2^5}=\dfrac{1}{3}$, $\therefore y=-1$

75. $9^{x+1}=16$, $3^{2x+2}=16$, $3^{2x}\cdot 3^2=16$, $(3^x)^2=\dfrac{16}{9}$, $\therefore 3^x=\dfrac{4}{3}$

76. $4^{2x}\cdot 2^{3y-5}=1$, $2^{4x}\cdot 2^{3y-5}=2^0$, $4x+3y-5=0\ldots\ldots(1)$;

$3^{2x} \cdot 9^{y-1} = 27$, $3^{2x} \cdot 3^{2y-2} = 3^3$, $2x + 2y - 2 = 3 \ldots\ldots(2)$;
Solving (1) and (2), we have $x = -2.5$, $y = 5$.

77. $= (\frac{1}{x} - \frac{1}{y})^{-2} = (\frac{y-x}{xy})^{-2} = \frac{x^2y^2}{(y-x)^2} = \frac{x^2y^2}{(x-y)^2}$

UNIT 3 NUMERAL SYSTEMS

1. D	2. B	3. A	4. C	5. C	6. B	7. D	8. A
9. B	10. A	11. A	12. C	13. C	14. D	15. D	16. B
17. A	18. D	19. B	20. A	21. D	22. C	23. B	24. B
25. B	26. A	27. C	28. D	29. A	30. C	31. B	32. A
33. B	34. A	35. D	36. A	37. C	38. D	39. B	40. B
41. C	42. D	43. A	44. B				

Explanatory Notes

42. $A9_{16} = 10 \times 16 + 9 \times 1 = 169_{10}$,
by continual division, $169_{10} = 10101001_2$, $\therefore$ $A9_{16} = 10101001_2$

43. $1101100_2 = 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2 + 0 \times 1$
$= 108_{10}$, by continual division, $108_{10} = 6C_{16}$, $\therefore$ $1101100_2 = 6C_{16}$

44. Difference = $(10b + a) - (10a + b) = 9b - 9a$

UNIT 4 FACTORIZATION OF SIMPLE POLYNOMIALS (3)

1. D	2. A	3. A	4. C	5. D	6. B	7. C	8. D
9. B	10. C	11. C	12. D	13. B	14. A	15. C	16. B
17. A	18. D	19. B	20. A	21. C	22. D	23. B	24. D
25. A	26. B	27. D	28. C	29. C	30. C	31. A	32. B
33. C	34. A	35. D	36. A	37. C	38. A	39. D	40. C
41. C	42. D	43. A	44. B	45. C	46. D	47. B	48. B
49. B	50. A	51. C	52. A	53. A	54. D	55. C	

Explanatory Notes

1. $\because$ Coefficient of $x = -b$ which is negative
and the constant term = c which is positive,
$\therefore$ we have $(x - p)(x - q) = x^2 - px - qx + pq = x^2 - (p+q)x + pq$

2. $\because$ Constant term = $pq = -c$ which is negative,
$\therefore$ either p or q is negative, i.e. I and II are not necessarily true.

$\because$ Coefficient of $x = p + q = b$ which is positive,
$\therefore$ $p + q > 0$, i.e. III is true. $\therefore$ The answer is B.

9. A. $x^2 + 17x + 60 = (x+12)(x+5)$;
B. $x^2 - 17x - 60 = (x-20)(x+3)$;
C. $x^2 + 17x - 60 = (x+20)(x-3)$;
D. $x^2 - 17x + 60 = (x-12)(x-5)$

11. I. $10y^2 - y - 2 = (5y+2)(2y-1)$;
II. $2 - y - 10y^2 = (2-5y)(1+2y)$;
III. $10y^2 - 9y + 2 = 2 - 9y + 10y^2 = (2-5y)(1-2y)$;
$\therefore$ The answer is C.

14. $x^4 - 5x^2 - 36 = (x^2-9)(x^2+4) = (x+3)(x-3)(x^2+4)$

15. $12k(k+1) - 5(k+2) = 12k^2 + 7k - 10 = (4k+5)(3k-2)$

16. $(7t+3)(2t+1) - 15 = 14t^2 + 7t + 6t + 3 - 15 = 14t^2 + 13t - 12$
$= (2t+3)(7t-4)$

21. A. $6x^2 + x - 7 = (6x+7)(x-1)$;
B. $6x^2 + 11x - 7 = (2x-1)(3x+7)$;
D. $6x^2 + 19x - 7 = (2x+7)(3x-1)$;
$\therefore$ The answer is C.

22. I. $5a^2 - 3a + 1 - 3 = 5a^2 - 3a - 2 = (5a+2)(a-1)$;
II. $5a^2 - 3a + 1 - 3a = 5a^2 - 6a + 1 = (5a-1)(a-1)$;
III. $5a^2 - 3a + 1 - 3a^2 = 2a^2 - 3a + 1 = (2a-1)(a-1)$;
$\therefore$ The answer is D.

29. $x^6 - 64 = (x^3+8)(x^3-8) = (x+2)(x^2-2x+4)(x-2)(x^2+2x+4)$
$\because$ $(x-2)(x+2) = x^2 - 4$, $\therefore$ The answer is C.

30. A. $a^6 - 1 = (a^3+1)(a^3-1) = (a+1)(a^2-a+1)(a-1)(a^2+a+1)$;
B. $a^4 - 1 = (a^2+1)(a^2-1) = (a^2+1)(a+1)(a-1)$;
C. $a^3 - 1 = (a-1)(a^2+a+1)$;
D. $a^2 - 1 = (a+1)(a-1)$

31. $= (2k+3)^3 + 5^3 = (2k+3+5)[(2k+3)^2 - (2k+3)(5) + 5^2]$
$= (2k+8)(4k^2 + 12k + 9 - 10k - 15 + 25) = 2(k+4)(4k^2 + 2k + 19)$

32. $= 1 + (2m-2)^3 = (1 + 2m - 2)[1 - (2m-2) + (2m-2)^2]$
$= (2m-1)(1 - 2m + 2 + 4m^2 - 8m + 4) = (2m-1)(4m^2 - 10 + 7)$

33. $= 3^3 - (2y+1)^3 = (3 - 2y - 1)[3^2 + 3(2y+1) + (2y+1)^2]$
$= (2-2y)(9 + 6y + 3 + 4y^2 + 4y + 1) = 2(1-y)(4y^2 + 10y + 13)$

36. $x^4 - 3x^2 - 4 = (x^2-4)(x^2+1) = (x+2)(x-2)(x^2+1)$

37. $y^4 - 10y^2 + 9 = (y^2-9)(y^2-1) = (y+3)(y-3)(y+1)(y-1)$

38. $a^6 + 5a^3 - 24 = \underline{(a^3+8)}(a^3-3) = (a+2)\underline{(a^2-2a+4)}\underline{(a^3-3)}$

39. $x^6-1=(x^3+1)(x^3-1)=(x+1)(x^2-x+1)(x-1)(x^2+x+1)$
or $x^6-1=(x^2)^3-1=(x^2-1)(x^4+x^2+1)=(x+1)(x-1)(x^4+x^2+1)$
$\therefore$ The answer is D.

40. $=1+(y^3)^3=(1+y^3)(1-y^3+y^6)=(1+y)(1-y+y^2)(1-y^3+y^6)$

41. $=x^3(x+1)-(x+1)=(x+1)(x^3-1)=(x+1)(x-1)(x^2+x+1)$

42. $=a(a^3-b^3)-b(a^3-b^3)=(a^3-b^3)(a-b)=(a-b)^2(a^2+ab+b^2)$

43. $=8x^3(4x^2-1)+(4x^2-1)=(4x^2-1)(8x^3+1)$
$=(2x+1)(2x-1)(2x+1)(4x^2-2x+1)$
$=(2x+1)^2(2x-1)(4x^2-2x+1)$

46. $=\dfrac{1}{(y+3)(y-2)}+\dfrac{1}{(y+3)(y+8)}=\dfrac{y+8+y-2}{(y+3)(y-2)(y+8)}$
$=\dfrac{2(y+3)}{(y+3)(y-2)(y+8)}=\dfrac{2}{(y-2)(y+8)}$

47. $=\dfrac{1}{(4-m)(1-m)}+\dfrac{2}{(4-m)(2+m)}=\dfrac{2+m+2(1-m)}{(4-m)(1-m)(2+m)}$
$=\dfrac{4-m}{(4-m)(1-m)(2+m)}=\dfrac{1}{(1-m)(2+m)}$

48. $y^3+\dfrac{1}{y^3}=(y+\dfrac{1}{y})(y^2-1+\dfrac{1}{y^2})=5(23-1)=110$

49. $a+b=2$, $(a+b)^2=4$, $a^2+2ab+b^2=4$, $2ab=4-8$,
$\therefore$ $ab=-2$; $\therefore$ $a^3+b^3=(a+b)(a^2-ab+b^2)=2[8-(-2)]=20$

50. Area = $56+10x-x^2=(4+x)(14-x)$,
$\therefore$ perimeter = $2[(4+x)+(14-x)]=36$ cm

51. $x^2+24x+80=(x+20)(x+4)$; $x+4=9$, $x=5$;
$\therefore$ The larger number = 5 + 20 = 25

52. $=m^4+n^4+m^3n+mn^3$ $=m^3(m+n)+n^3(m+n)$ $=(m+n)(m^3+n^3)$
$=(m+n)^2(m^2-mn+n^2)$

53. $=[(a^2-2a)+1]^2=[(a-1)^2]^2=(a-1)^4$

54. $=[(x^2+3x)+2][(x^2+3x)-10]=(x+1)(x+2)(x+5)(x-2)$

55. $=(x^2+4x+4)-(y^2-2y+1)=(x+2)^2-(y-1)^2$
$=[(x+2)+(y-1)][(x+2)-(y-1)]=(x+y+1)(x-y+3)$

UNIT 5 LINEAR INEQUALITIES IN ONE UNKNOWN

1. D	2. A	3. D	4. D	5. B	6. A	7. D	8. D
9. A	10. C	11.C	12. B	13. B	14. A	15. C	16. B
17. A	18. A	19. D	20. C	21. A	22. D	23. D	24. B

25. C	26. A	27. B	28. B	29. A	30. D	31. C	32. B
33. B	34. D	35. D	36. A	37. B	38. B	39.A	40. C
41. D	42.D	43. C	44. B	45. C	46. B	47. A	48. A
49. B	50. A	51. C	52. D	53. A	54. D	55. A	56. C
57.C	58. D	59. C	60. C	61. C	62. D	63. C	64. C
65. B	66. D	67. B	68. D				

Explanatory Notes

9. When $x=-1$, $y=-3$, $z=-9$,
 $\because\ -1-(-3)=2<6=-3-(-9)$, $\therefore$ II is not true.
 $\because\ 1=(-1)^2<(-9)^2=81$, $\therefore$ III is not true.

13. $\because\ \frac{m}{-5}$ is positive and $\frac{n}{5}$ is negative, $\therefore$ II is true.

29. $7x-4y<-1$, $7x+1<4y$, $\therefore y>\frac{7x+1}{4}$

34. Larger number = x, smaller number = $x-2$; $x+(x-2)\geq 30$,
 $2x\geq 32$, $x\geq 16$. $\because$ x is odd, $\therefore$ minimum value = 17

35. Smaller number = x, larger number = $x+4$; $x>\frac{x+4}{2}$, $2x>x+4$,
 $x>4$. $\because$ x is a multiple of 4, $\therefore$ least value = 8

41. Selling price of each apple = \$$x$; $\frac{(80-20)x-80}{80}\times 100\%\geq 20\%$,
 $\frac{60x-80}{80}\geq\frac{1}{5}$, $60x-80\geq 16$, $x\geq 1.6$. $\therefore$ Minimum price = \$1.6

42. $\because\ x<-3$, $\therefore\ x-1<-4<-3<-2$, $\therefore$ I, II and III are true.

43. $\because\ x\geq 15$, $\therefore\ x+1\geq 16$.
 I is true because 16 > 15; II is not true when x = 15;
 III is not true because 16 < 17.

45. $y>-5$, $1-\frac{x}{3}>-5$, $-\frac{x}{3}>-6$, $x<18$. $\because$ x is non-negative,
 $\therefore$ no. of possible values = 18 (from 0 to 17 inclusive).

46. $2a-b+10=0$, $2a=b-10$, $a=\frac{b-10}{2}$; $\because\ a\leq 0$, $\therefore\ \frac{b-10}{2}\leq 0$,
 $b-10\leq 0$, $b\leq 10$. $\therefore$ Greatest value = 10

47. I. $\because 4a<a<b$, $\therefore$ true.
 II. $\because -4b>0>a$, $\therefore$ true.
 III. When $a=-3$, $b=-2$, $-3>-8=4(-2)$, $\therefore$ not true.
 $\therefore$ The answer is A.

48. When m = 1.5, n = 1, $\because\ 1.5-1=0.5<1$, $\therefore$ A is not always true.

49. I. $\because a>0$ and $a>b$, $\therefore a^2>ab$, $\therefore$ true.
 II. $\because a^3$ is positive and b^3 is negative, $\therefore\ a^3>b^3$, $\therefore$ true.

III. When $a = 1$, $b = -4$, $1^2 = 1 < 16 = (-4)^2$, $\therefore$ not true.

$\therefore$ The answer is B.

51. $\because ab < c$, $\therefore ab - c < 0 < 1$

54. I and II are not true when x is negative.
III is not true when $0 < x < 1$.

56. I. When $m = -4$, $n = 24$, $\dfrac{24}{-4} = -6 < -3$, $\therefore$ not true.

II. $m < -3$, $mn < -3n$ ……(i); $n > 9$, $-3n < 27$ ……(ii);
Combining (i) and (ii), we have $mn < -27$, $\therefore$ true.

III. $\because$ $m < -3$ and $n > 9$, $\therefore m^2 > 9$ and $n^2 > 81$, $\therefore m^2 + n^2 > 90$,
$\therefore$ true.

$\therefore$ The answer is C.

57. I. If y is a positive integer, $\dfrac{1}{y}$ is a proper fraction less than or equal to 1, i.e. $\dfrac{1}{y} \le 1 < 10$, $\therefore$ true.

II. $\because$ $\dfrac{1}{y}$ is negative when y is negative, $\therefore$ $\dfrac{1}{y} < 0 < 10$, $\therefore$ true.

III. When $y = \dfrac{1}{20}$, $1 \div (\dfrac{1}{20}) = 20 > 10$, $\therefore$ not true.

$\therefore$ The answer is C.

60. $(1-\sqrt{3})x < 2$, $x > \dfrac{2}{1-\sqrt{3}}$ ($\because 1-\sqrt{3}$ is negative), $x > \dfrac{2(1+\sqrt{3})}{1-3}$,
$x > \dfrac{2(1+\sqrt{3})}{-2}$, $\therefore x > -1-\sqrt{3}$

61. $ay + 9a \le 2y - a$, $10a \le 2y - ay$, $(2-a)y \ge 10a$,
$\therefore$ $y \ge \dfrac{10a}{2-a}$ ($\because a < 2$)

62. $mx + m^2 > nx + n^2$, $mx - nx > n^2 - m^2$, $(m-n)x > (n-m)(n+m)$,
$x < \dfrac{(n-m)(n+m)}{m-n}$ ($\because$ $m < n$), $\therefore x < -m - n$

64. Smallest possible value = $-3 - (-1) = -2$

66. Greatest possible value = $(-8)^2 + (-3)^2 = 73$

67. Smallest possible value = $(-8)(2) = -16$;
greatest possible value = $(-8)(-3) = 24$; $\therefore -16 \le ab \le 24$

68. Greatest possible value = $\dfrac{-3}{-1} = 3$

UNIT 6 PERCENTAGES (2)

1. C	2. D	3. A	4. A	5. B	6. D	7. A	8. C
9. C	10. C	11. D	12. A	13. B	14. A	15. A	16. D
17. C	18. D	19. D	20. C	21. D	22. C	23. A	24. B
25. B	26. B	27. C	28. B	29. C	30. D	31. A	32. A
33. A	34. A	35. B	36. C	37. B	38. A	39. D	40. C
41. A	42. B	43. B	44. A	45. B	46. D		

Explanatory Notes

8. Amount $= 5000(1+4\%\times 2+5\%\times 3) = \$6150$

17. Compound interest $= 18000(1+\frac{2\%}{4})^4(1+\frac{2.8\%}{4})^8 - 18000 = \$1416.6$

18. $P[(1+6\%)^2-1]\geq 4000$, $0.1236P\geq 4000$, $P\geq 32362.46$.
$\because$ P is a multiple of 10, $\therefore$ $P = 32370$

19. Difference $= 90000[(1+\frac{9\%}{12})^{18}-1] - 90000\times 9\%\times\frac{18}{12}$
$= 12956.4 - 12150$ $= \$806.4$

20. Amount owed after the 1st payment $= 95000(1+\frac{15\%}{12}) - 25000$
$= \$71187.5$, amount owed after the 2nd payment
$= 71187.5(1.0125) - 25000$ $= \$47077$

21. Amount owed at the end of 1st month $= 18000(1+\frac{24\%}{12}) = \$18360$,
amount owed at the end of 2nd month $= (18360-5000)(1.02)$
$= \$13627.2$,
amount owed at the end of 3rd month $= (13627.2-5000)(1.02)$
$= \$8800$

22. Amount owed after 2 months $= 26000(1+\frac{16\%}{12})^2 - 6000$
$= \$20697.96$,
amount owed after 4 months $= 20697.96(1+\frac{16\%}{12})^2 - 6000$
$= \$15254$

23. Interest = amount in 4 years – amount in 3 years
$= 44000(1+\frac{8\%}{2})^8 - 44000(1+\frac{8\%}{2})^6$
$= 60217.04 - 55674.04$ $= \$4543$

32. Decay factor = r; $32000r^2 = 23120$, $r = \sqrt{\frac{23120}{32000}} = 0.85$;
$\therefore$ Value in 2006 $= 23120(0.85)^5 = \$10258$

33. Increase in book collection $= 74000[(1+8\%)^3(1+5\%)^2 - 1]$
$= 28774$

34. Sales figure $= 35000 \div (1+4\%)^4(1+8\%)^6 = 18854$

35. Let principal = $\$P$, no. of years = n. $P(1+10\%n) = P(1+150\%)$,
$1 + 0.1, n = 2.5$, $\therefore n = 15$

36. Let principal = $\$P$, interest rate = r. $P(1+16r) = 2P$, $1 + 16r = 2$,
$\therefore r = 0.0625 = 6.25\%$

37. Principal $= 3200 \div (1+6\% \times 4\frac{2}{3}) = \$2500$,
$\therefore$ required amount $= 2500(1+3\frac{1}{3}\% \times 6) = \$3000$

38. $\because 6\% \times 5 = 2.4\% \times 12.5 = 0.3$, $\therefore$ the answer is A.

39. A. $(1+\frac{18\%}{12})^{12} = 1.1956$; B. $(1+\frac{18.2\%}{4})^4 = 1.1948$;
C. $(1+\frac{18.8\%}{2})^2 = 1.1968$; D. $(1+19\%) = 1.19$;
$\therefore$ Kelvin should choose C.

40. Let principal = $\$P$. $P[(1+r\%)^3 - 1] = P \times 20\%$, $(1+r\%)^3 = 1.2$,
$1 + r\% = \sqrt[3]{1.2} = 1.063$, $r\% = 0.063$, $\therefore r = 6.3$

41. Interest earned in the 3rd year
$= 65000[(1+\frac{4\%}{4})^{12} - (1+\frac{4\%}{4})^8] = \$2857.9$,
interest earned in the 2nd year $= 65000[(1.01)^8 - (1.01)^4] = \$2746.4$,
$\therefore$ difference $= 2857.9 - 2746.4 = \$112$

42. Amount at the end of 2003
$= 6800(1+5\%)^4 + 6800(1+5\%)^3 + 6800(1+5\%)^2 + 6800(1+5\%)$
$= \$30774$

43. Let monthly installment = $\$x$. $[7000(1+\frac{12\%}{12}) - x](1+\frac{12\%}{12}) - x = 0$,
$(7070 - x)(1.01) - x = 0$, $7140.7 - 1.01x - x = 0$, $2.01x = 7140.7$,
$\therefore x = 3552.6$

44. Let annual deposit = $\$x$. $x(1+6\%)^3 + x(1+6\%)^2 + x = 300000$,
$x[(1.06)^3 + (1.06)^2 + 1.06 + 1] = 300000$, $\therefore x = 68577$

45. Decrease in value $= 95000(1-12\%)^4 - 95000(1-12\%)^5$
$= 56971.06 - 50134.53 = \$6837$

46. Value $= 16000(1+5\%)^2(1-10\%)^5 = \$10416$

UNIT 7 PERCENTAGES (3)

1. B	2. C	3. D	4. C	5. C	6. C	7. A	8. B
9. C	10. D	11. A	12. B	13. C	14. A	15. A	16. B
17. D	18. A	19. A	20. C	21. B	22. C	23. D	24. A
25. D	26. C	27. B	28. A	29. C	30. A	31. B	32. D
33. A	34. A	35. A	36. B	37. D	38. C	39. B	40. B
41. C	42. A	43. D	44. C	45. B	46. A	47. B	48. C
49. C	50. C	51. B	52. C	53. B	54. A		

Explanatory Notes

3. Difference $=(1700\times 4)\div 5\%=\$136000$

6. Difference $=(7500\times 12)\times 14\%\times 80\%\times 15\%=\$1512$

9. Gross profit $=345600\div 16\%\div (1-\frac{2}{3})=\$6480000$

10. Gross profits $=81300\times (12+3)=\$1219500$,
operating expenses $=54700\times 12=\$656400$,
$\therefore$ profits tax $=(1219500-656400)\times 16\%=\$90096$

15. Maximum income = Total allowances = \$100000

20. Let the side of small cube $= x$.
total surface area of original cube $=6(3x)^2=54x^2$;
total surface area of small cubes $=6x^2\times 27=162x^2$;
$\therefore$ percentage increase $=\dfrac{162x^2-54x^2}{54x^2}\times 100\%=200\%$

23. Let x kg be the weight before joining the slimming programme.
The required percentage change $=\dfrac{x-(0.7x)(1.2x)}{(0.7x)(1.2x)}\times 100\%=+19\%$

24. Original volume $=\pi r^2 h$, new volume $=\pi[r(1+x\%)]^2[h(1-20\%)]$;
$\pi r^2 h=\pi[r(1+x\%)]^2[h(1-20\%)]$, $\pi r^2 h=\pi r^2 h(1+x\%)^2(0.8)$,
$0.8(1+x\%)^2=1$, $(1+x\%)^2=1.25$, $1+x\%=1.118$, $x=11.8$.
$\therefore$ Percentage increase in radius = 11.8%

25. $A=C(1-15\%)=0.85C$, $B=C(1+10\%)=1.1C$,
$\therefore$ required percentage $=\dfrac{B}{A}\times 100\%=\dfrac{1.1C}{0.85C}\times 100\%=129.4\%$

26. $Q=R\times 120\%=1.2R$, $Q=S\times 75\%=0.75S$;
I. $\dfrac{Q-R}{Q}\times 100\%=\dfrac{1.2R-R}{1.2R}\times 100\%=16.6\%$
II. $\dfrac{S-Q}{S}\times 100\%=\dfrac{S-0.75S}{S}\times 100\%=25\%$

III. $\dfrac{S-R}{S}\times 100\% = \dfrac{\frac{1}{0.75}Q-\frac{1}{12}Q}{\frac{1}{0.75}Q}\times 100\% = 37.5\%$

$\therefore$ The answer is C.

27. Percentage change $=[(1+20\%)(1-15\%)-1]\times 100\% = 2\%$

28. Percentage of failed students $=60\%(1-55\%)=27\%$,
percentage of students who passed $=1-27\%=73\%$

I. Percentage $=\dfrac{73}{27}\times 100\% = 270\%$

II. Percentage $=\dfrac{73-27}{27}\times 100\% = 170\%$

III. Percentage $=\dfrac{73-27}{73}\times 100\% = 63\%$

$\therefore$ The answer is A.

30. Percentage change
$=[0.25(1+30\%)+0.55(1-10\%)+0.2(1+5\%)-1]\times 100\% = 3\%$

32. Suppose x g of sugar should be added.
$\dfrac{400\times 15\%+x}{400+x}\times 100\% = 20\%$, $\dfrac{60+x}{400+x}=\dfrac{1}{5}$, $300 + 5x = 400 + x$,
$4x = 100$, $\therefore$ $x = 25$

35. $\because$ Rates are paid quarterly, $\therefore$ on 31/12/2005, the man needed to pay $(214000+348000)\times 5\% \div 4 = \$7025$

36. Increase $=(3450\div 15\%\div 80\%)\div 12 = \$2396$

38. Net profits $=63360\div 16\% = \$396000$,
$\therefore$ gross profits $=396000+396000\times 120\% = \$871200$

39. Operating expenses $=46800\div 5\% - 46800\div 15\% = \$624000$

40. Let profits tax = \$$x$, then gross profits $=x\div 6\% = \dfrac{x}{0.06}$,
net profits $=x\div 15\% = \dfrac{x}{0.15}$.

$\therefore$ Required percentage $=\dfrac{\frac{x}{0.06}-\frac{x}{0.15}}{\frac{x}{0.06}}\times 100\% = 60\%$

41. Salaries tax on the 1st \$90000 $=30000\times(2\%+8\%+14\%)$ = \$7200,
$\therefore$ total income $=90000+7200\div 20\%+100000 = \$226000$

42. Using progressive rate:
Net chargeable income $=2000000-100000-30000\times 2$
$= \$1840000$,
$\therefore$ salaries tax
$=30000\times(2\%+8\%+14\%)+(1840000-90000)\times 20\% = \$320000$
Using standard rate:
Salaries tax = $2000000\times 16\% = \$320000$
$\therefore$ Salaries tax is \$320000 which is lower.

43. Let total income = \$$x$.
$30000\times(2\%+8\%+14\%)+(x-200000-90000)\times20\% = x\times16\%$,
$7200+0.2(x-290000)=0.16x$, $0.04x=50800$, $\therefore\ x=1270000$

44. Let number of members in 2000 = x. $x(1+10\%)^3(1-5\%)^2\le900$,
$x\le749.2$. $\therefore$ Maximum number = 749

45. Let cost = \$$c$ and selling price = \$$s$, then $9c = 6s$ or $s = 1.5c$.
$\therefore$ Profit percentage $=\dfrac{s-c}{c}\times100\%=\dfrac{1.5c-c}{c}\times100\%=50\%$

46. $D=E(1+25\%)=1.25E$, $F=D(1-16\%)=0.84D$;
$\because\ \dfrac{F-E}{E}\times100\%=\dfrac{0.84D-\frac{D}{1.25}}{\frac{D}{1.25}}\times100\%=5\%$,
$\therefore$ A is true but C is false.
$\because\ \dfrac{F-E}{F}\times100\%=\dfrac{0.84D-\frac{D}{1.25}}{0.84D}\times100\%=4.76\%$,
$\therefore$ B and D are false.

47. Let number = N. $N(1+r\%)(1-r\%)=N(1-36\%)$, $1-(r\%)^2=0.64$,
$(r\%)^2=0.36$, $r\%=0.6$, $\therefore\ r=60$

48. Let original income = \$$x$, then original savings $=x\times20\%=0.2x$,
new savings $=x(1+15\%)-x(1-20\%)(1+10\%)=0.27x$.
$\therefore$ Percentage change $=\dfrac{0.27x-0.2x}{0.2x}\times100\%=35\%$

49. Original price per kg $=60\times\dfrac{3}{10}+32\times\dfrac{7}{10}=\$40.4$,
new price per kg $=60(1-15\%)\times\dfrac{3}{10}+32(1+25\%)\times\dfrac{7}{10}=\$43.3$,
$\therefore$ percentage change $=\dfrac{43.3-40.4}{40.4}\times100\%=7.2\%$

50. Let distance = D and speed = S,
then original time $=\dfrac{D}{S}$, new time $=\dfrac{D}{S(1-50\%)}=\dfrac{D}{0.5S}$.
$\therefore$ Percentage increase $=\dfrac{\frac{D}{0.5S}-\frac{D}{S}}{\frac{D}{S}}\times100\%=100\%$

51. Let original price per kg = \$$x$. $\dfrac{480}{x}-\dfrac{480}{x(1+20\%)}=10$,
$1.2(480)-480=10(1.2x)$, $96=12x$, $\therefore\ x=8$

52. Let number of articles = n. $\dfrac{600}{n}(1+15\%)(n-5)-600=21$,
$600(1.15)(n-5)=621n$, $690n-3450=621n$, $\therefore\ n=50$

53. Let cost of X = a, then cost of Y = $a(1 + 25\%) = 1.25a$,
total cost = $a + 1.25a = 2.25a$. If profit percentage on Y = $r\%$,
then total selling price $=a(1+60\%)+1.25a(1+r\%)$,

$\therefore$ $(2.25a)(1+50\%) = a(1+60\%)+(1.25a)(1+r\%)$,
$1.775a = 1.25a(1+r\%)$, $1+r\% = 1.42$, $r\% = 0.42$, $r = 42$.
$\therefore$ Profit percentage on Y = 42%

54. Let profit percentage of remaining stock = r%.
$[\frac{1}{2}(1+20\%)+\frac{1}{6}(1-16\%)+(1-\frac{1}{2}-\frac{1}{6})(1+r\%)-1]\times 100\% = 15\%$,
$0.6+0.14+\frac{1}{3}(1+r\%)-1 = 0.15$, $\frac{1}{3}(1+r\%) = 0.41$, $r\% = 0.23$,
$r = 23$. $\therefore$ Profit percentage = 23%

UNIT 8 MORE ABOUT DEDUCTIVE GEOMETRY

1. B	2. C	3. C	4. A	5. D	6. A	7. C	8. C
9. C	10. A	11. A	12. D	13. D	14. B	15. C	16. A
17. D	18. A	19. B	20. C	21. C	22. C	23. A	24. D
25. C	26. B	27. B	28. B	29. D	30. A	31. A	32. C
33. B	34. C	35. D	36. D	37. A	38. B	39. B	40. C
41. D	42. A	43. D	44. D	45. C	46. A	47. D	48. B
49. D	50. C	51. D	52. A	53. C	54. A	55. C	56. C

Explanatory Notes

14. $\because \Delta ABD \sim \Delta BCD$, $\therefore \frac{AD}{BD} = \frac{BD}{CD}$, $BD^2 = AD \cdot CD = 18 \cdot 32 = 576$,
$\therefore BD = \sqrt{576} = 24$ cm

20. $\because \Delta QRU \sim \Delta TSU$ (AAA), $\therefore \frac{QR}{TS} = \frac{QU}{TU} = \frac{4}{10} = \frac{2}{5}$.
$\because \Delta PQR \sim \Delta PST$ (AAA), $\therefore \frac{PQ}{PS} = \frac{QR}{TS}$, $\frac{y}{y+9} = \frac{2}{5}$, $5y = 2y + 18$,
$3y = 18$, $\therefore y = 6$

28. $OP = OQ = OR = 5$ (radii of circumcircle),
$\therefore OP^2 + OQ^2 + OR^2 = 5^2 + 5^2 + 5^2 = 75$

34. I. $\angle B = \angle ACB$, $\angle AFE = \angle B + \angle D$,
$\angle AEF = \angle CED = \angle ACB - \angle D = \angle B - \angle D$
$\therefore$ $\angle AFE > \angle AEF$, $\therefore AE > AF$

II. $\angle DCE$ and $\angle CED$ are obtuse and acute respectively,
$\therefore DE > CD$

III. If $\angle B = \angle BFD$, then $BD = DF$.
$\therefore$ The answer is C.

36. A. $CD = CE$, $CB = CA$, $\angle DCB = \angle ECA = 60°$,
$\therefore \Delta ACE \cong \Delta BCD$ (SAS)

B. $\because \Delta ACE \cong \Delta BCD$, $\therefore \angle AEC = \angle BDC = 90°$,
$\therefore \angle AEB = 180° - 90° = 90°$,
but $AB = AC$ and AE is common, $\therefore \Delta ACE \cong \Delta ABE$ (RHS)

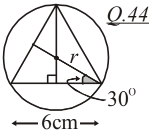
C. $\angle DBC = 180° - \angle BCD - \angle BDC = 180° - 60° - 90° = 30°$,
$\angle BDE = 90° - 60° = 30°$,
$\therefore BE = DE$, $\therefore \Delta BDE$ is isosceles.

37. I. $\Delta ABC \cong \Delta CDE$ (ASA/AAS)

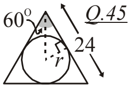
II. $AC = EC$, $AF = EF$, $CF = CF$, $\therefore \Delta AFC \cong \Delta EFC$ (SSS),
$\therefore \angle AFC = \angle EFC = 90°$

III. $\because \Delta AFC \cong \Delta EFC$, $\therefore \angle ACF = \angle ECF = \dfrac{90°}{2} = 45°$,
$\therefore \angle FAC = 180° - 90° - 45° = 45°$,
$\therefore AF = FC$, but $AB \neq BC$,
$\therefore \Delta AFC$ is not congruent to ΔABC.
$\therefore$ The answer is A.

39. $\because \Delta ABC \sim \Delta EDC$ (AAA), $\therefore \dfrac{CD}{21} = \dfrac{40}{20} = 2$, $CD = 42$.
$\because CE^2 + CD^2 = 40^2 + 42^2 = 3364 = 58^2 = DE^2$, $\therefore \angle DCE = 90°$.
$\sin m = \dfrac{40}{58}$, $\therefore m = 43.6°$



44. $\dfrac{3}{r} = \cos 30° = \dfrac{\sqrt{3}}{2}$, $r = \dfrac{6}{\sqrt{3}} = 2\sqrt{3}$,
$\therefore$ area $= \pi(2\sqrt{3})^2 = 12\pi \text{ cm}^2$



45. $\tan(\dfrac{60°}{2}) = \dfrac{r}{12}$,
$\therefore r = 12\tan 30° = 12 \times \dfrac{\sqrt{3}}{3} = 4\sqrt{3}$ cm

47. $\because \Delta DNF \sim \Delta EMF$ (AAA), $\therefore \dfrac{MF}{NF} = \dfrac{EF}{DF}$, $\dfrac{MF}{6} = \dfrac{12}{10}$,
$\therefore MF = \dfrac{6}{5} \times 6 = 7.2$ cm

48. $ME = \sqrt{12^2 - 7.2^2} = 9.6$. $\because \Delta EHN \sim \Delta EFM$ (AAA),
$\therefore \dfrac{HN}{FM} = \dfrac{EN}{EM}$, $\dfrac{HN}{7.2} = \dfrac{6}{9.6}$, $\therefore HN = \dfrac{6}{9.6} \times 7.2 = 4.5$ cm

51. I and II. $AD + BD > AB$ ……(1);
$AD + CD > AC$ ……(2); $BD + CD > BC$ ……(3);
(1) + (2) + (3), $2(AD + BD + CD) > AB + BC + AC$,
$\therefore AD + BD + CD > \dfrac{1}{2}(AB + BC + AC)$

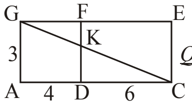
III. $\angle ADB = \angle BDC = \angle ADC = 360° \div 3 = 120°$,
$\therefore \angle ADB > \angle BAD \Rightarrow AB > BD$, $\angle ADC > \angle ACD \Rightarrow AC > AD$,
$\angle BDC > \angle CBD \Rightarrow BC > CD$,
$\therefore AB + AC + BC > BD + AD + CD$
$\therefore$ The answer is D.

52. $\because \Delta FCD \sim \Delta FAB$, $\therefore \dfrac{FD}{FB} = \dfrac{k}{30}$; $\because \Delta BCD \sim \Delta BEF$, $\therefore \dfrac{BD}{BF} = \dfrac{k}{20}$;
$\dfrac{FD}{FB} + \dfrac{BD}{BF} = \dfrac{k}{30} + \dfrac{k}{20}$, $\dfrac{FD + BD}{BF} = \dfrac{5k}{60}$, $\dfrac{k}{12} = \dfrac{BF}{BF} = 1$, $\therefore k = 12$

53. $\because \dfrac{AB}{AD} = \dfrac{AC}{AE} = \dfrac{AD}{AF}$, $\therefore \dfrac{12}{AD} = \dfrac{AD}{27}$, $AD^2 = 324$, $\therefore AD = 18$

54. $\because \Delta SRQ \sim \Delta TSR$ (AAA), $\therefore \dfrac{RS}{ST} = \dfrac{QR}{RS}$, $RS^2 = 18 \cdot 8 = 144$,
$\therefore RS = 12$

55. $\because \Delta PRQ \sim \Delta QSR$ (AAA), $\therefore \dfrac{PR}{QS} = \dfrac{QR}{RS}$, $\dfrac{PR}{18} = \dfrac{18}{12}$,
$PR = \dfrac{18}{12} \times 18 = 27$. $\therefore PS = 27 - 12 = 15$



Q.56

56. By flattening the two walls, the length of wire is minimum when *CKG* is a straight line.
$\because \Delta CKD \sim \Delta CGA$, $\therefore \dfrac{DK}{3} = \dfrac{6}{6+4} = \dfrac{3}{5}$, $\therefore DK = \dfrac{3}{5} \times 3 = 1.8$ m

UNIT 9 QUADRILATERALS

1. D	2. B	3. C	4. B	5. D	6. B	7. C	8. A
9. A	10. C	11. A	12. D	13. B	14. C	15. B	16. D
17. C	18. D	19. A	20. B	21. B	22. A	23. C	24.D
25. C	26. B	27. B	28. C	29. C	30. A	31. B	32. A
33. D	34. C	35. C	36. D	37. A	38. B	39. D	40. D
41. B	42. A	43. C	44. D	45. D	46. B	47. B	48. C
49. D	50. B	51. B	52. A				

Explanatory Notes

13. $x + 8 = 3y$, $x - 3y + 8 = 0$ ……(1);
$4x - y = 9 - x$, $5x - y - 9 = 0$ ……(2);
Solving (1) and (2), we have $x = 2.5$, $y = 3.5$.

15. $\angle ECB = 60° + 90° = 150°$;
$\because EC = CD = CB$, $\therefore \angle CBE = (180° - 150°) \div 2 = 15°$,
$\therefore \angle AFE = \angle CFB = 180° - \angle ACB - \angle CBE = 180° - 45° - 15°$
$= 120°$

16. I. $\because \Delta SKU \cong \Delta SKT$ (RHS/AAS),
$\therefore \angle KSU = \angle KST = 60° \div 2 = 30°$,
but $\angle RST = 90° - 60° = 30°$, $\therefore \angle KST = \angle RST$,
also $\angle SKT = \angle R = 90°$ and ST is common,
$\therefore \Delta RST \cong \Delta KST$ (AAS)
II. $\angle SKU = \angle QKT$, $KU = KT$, $\angle SUK = \angle QTK$,
$\therefore \Delta SKU \cong \Delta QKT$ (ASA), $\therefore$ SK = QK
III. $\because PS = QR$ and $SU = QT$, $\therefore PU = RT$
$\therefore$ The answer is D.

17. $QR = PS = 13$, $ST = \frac{1}{2}QS = \frac{1}{2} \times \sqrt{13^2 - 5^2} = \frac{1}{2} \times 12 = 6$,
$\therefore PR = 2RT = 2\sqrt{5^2 + 6^2} = 2\sqrt{61} = 15.6$ cm

24. Area of ΔAPQ : area of $PQCB = 1 : 3$

27. $\frac{a-1}{3} = \frac{CD}{DE} = \frac{GF}{FE} = \frac{8}{a+1}$, $(a-1)(a+1) = 24$, $a^2 - 1 = 24$,
$a^2 = 25$, $\therefore a = 5$

28. $\frac{y}{3} = \frac{AC}{CE} = \frac{y+3}{8}$, $8y = 3y + 9$, $5y = 9$, $\therefore y = 1.8$

29. $BG = \frac{1}{2}CE$, $BF = 2CE$, $\therefore BG : BF = \frac{1}{2}CE : 2CE = 1 : 4$

34. $3y + 2 = x + y$, $x - 2y = 2$ ……(1);
$2x - 4 = x + y$, $x - y = 4$ ……(2);
Solving (1) and (2), we have $x = 6$, $y = 2$.
$\therefore$ Area $= \frac{1}{2}(6+2)^2 \times 4 = 128$ sq. units

35. Let $WZ = YZ = a$. $\because \Delta WZK \sim \Delta HYK$ (AAA),
$\therefore \frac{WZ}{HY} = \frac{ZK}{YK}$, $\frac{a}{9} = \frac{a-6}{6}$, $6a = 9a - 54$, $3a = 54$, $\therefore a = 18$

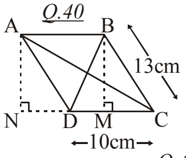
36. $\angle EBA = \angle EAB = 55°$, $\angle DEA = 55° + 55° = 110°$,
$\angle AEF = 110° - 60° = 50°$, but $AE = DE = EF$,
$\therefore \angle AFE = (180° - 50°) \div 2 = 65°$

37. I. $\angle DGH = \angle EGH = 90°$, $\angle HDG = \angle DEG = 90° - \angle EDG$,
$\therefore \Delta DHG \sim \Delta EDG$ (AAA)
II. $BC = DC$, $\angle BCH = \angle DCF = 90°$, $\angle CBH = \angle DEG = \angle FDC$,
$\therefore \Delta BHC \cong \Delta DCF$ (ASA)
III. $\because \Delta GEF \cong \Delta GBF$ and $\Delta HBC \cong \Delta CDF$, but ΔHBC is not congruent to ΔGBF, $\therefore \Delta CDF$ is not congruent to ΔGEF.

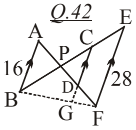
38. $BD = DE = BF = \sqrt{12^2 + 12^2} = 12\sqrt{2}$, $CH = FC = 12\sqrt{2} - 12$,
$DH = 12 - (12\sqrt{2} - 12) = 24 - 12\sqrt{2}$,
$DG = \frac{1}{2}DF = \frac{1}{2}\sqrt{12^2 + (12\sqrt{2} - 12)^2} = 6.494$,
$\therefore GH = \sqrt{(24 - 12\sqrt{2})^2 - 6.494^2} = 2.69$ cm

39. I. Size of each $\angle$ of pentagon = $\frac{(5-2)\times 180°}{5} = 108°$,
$\angle EAD = (180° - 108°) \div 2 = 36°$, $\angle FAB = 108° - 36° = 72°$,
but $\angle ABF = 108° \div 2 = 54°$,
$\therefore \angle AFB = 180° - 72° - 54° = 54°$,
$\therefore AB = AF$ and ΔABF is isosceles.
II. $\because \Delta ABF \cong \Delta CBF$ (SAS), $\therefore CF = AF = AB = CD$,
$\therefore \Delta CDF$ is isosceles.
III. $\because AB = AF = CB = CF$, $\therefore ABCD$ is a rhombus.

40. $CM = DM = 10 \div 2 = 5$,
$\therefore AN = BM = \sqrt{13^2 - 5^2} = 12$
but $CN = 10 + 5 = 15$,
$\therefore AC = \sqrt{12^2 + 15^2} = 19.2$ cm

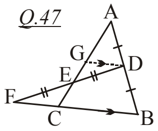


42. $\because AB // CD // EF$ and $BC = CE$,
$\therefore AD = DF$ and $BG = GF$,
$\therefore CG = 28 \div 2 = 14$ and $DG = 16 \div 2 = 8$,
$\therefore CD = 14 - 8 = 6$



45. A. $\because AF = FC$ and $CE // FG$, $\therefore AG = GE$, $\therefore CE = 2FG$,
$\therefore CD = 2CE = 4FG$
B. $\because \Delta DEH \sim \Delta FGH$ (AAA), $\therefore \frac{FH}{DH} = \frac{FG}{DE} = \frac{1}{2}$, $\therefore DH = 2FH$,
$\therefore BD = 2DF = 2(DH + FH) = 2(2FH + FH) = 6FH$
C. $\frac{GH}{EH} = \frac{FG}{DE} = \frac{1}{2}$, $\therefore EH = 2GH$,
$\therefore AE = 2EG = 2(EH + GH) = 2(2GH + GH) = 6GH$
D. $\tan \angle AED = \frac{AD}{DE} = 2$, $\angle AED = 63.4°$,
$\therefore \angle DHE = 180° - 45° - 63.4° = 71.6°$,
$\therefore \Delta DEH$ is not isosceles.

47. Draw $GD // FB$. $\because \Delta CEF \cong \Delta GED$ (ASA),
$\therefore CE = GE$. $\because AD = DB$ and $GD // CB$,
$\therefore AG = GC$. $\therefore AE : EC = 3 : 1$



48. $\frac{AD}{16} = \frac{18}{12} = \frac{3}{2}$, $AD = 24$. $\because \Delta ABG \sim \Delta DCG$ (AAA),

$\therefore \dfrac{DA}{AG}=\dfrac{CD}{BA}=\dfrac{10}{20}=\dfrac{1}{2}$, $\therefore$ AG = $24\times\dfrac{2}{2+1}$ = 16

49. $\because \Delta CDG \sim \Delta FGH$ (AAA), $\therefore \dfrac{FG}{CD}=\dfrac{GH}{DG}$,

but $\dfrac{GH}{DG}=\dfrac{EH}{AE}=\dfrac{9}{15}=\dfrac{3}{5}$, $\therefore \dfrac{FG}{10}=\dfrac{3}{5}$, $\therefore FG = 6$

51. Draw EG $\perp$ CD. $\because \Delta DEG \cong \Delta CEG$ (R.H.S.), $\therefore$ DG = CG.

$\because$ AD // EG // BC and DG = CG, $\therefore$ BE = EF (intercept thm.).

52. Let $\angle GAS = \angle DAS = a$ and $\angle FDS = \angle ADS = b$.

$\angle GAS + \angle DAS + \angle FDS + \angle ADS = 180°$,

$2a + 2b = 180°$, $a + b = 90°$,

$\therefore \angle DSA = 180° - (\angle DAS + \angle ADS) = 180° - (a + b) = 90°$,

$\therefore \angle PSR = \angle DSA = 90°$. Similarly, $\angle PQR = 90°$.

$\angle DEA = 180° - \angle EDA - \angle DAE = 180° - a - 2b$

$= 180° - (a + b) - b = 90° - b = a$,

but $\angle DCG = \dfrac{1}{2}\angle DCB = \dfrac{1}{2}\angle DAB = a$, $\therefore AE \parallel GC$,

$\therefore \angle SRQ = \angle SPQ = 90°$. $\because PS \neq SR$, $\therefore PQRS$ is a rectangle.

UNIT 10 STUDY OF 3-D FIGURES

1. D	2. B	3. C	4. C	5. A	6. B	7. D	8. A
9. D	10. A	11. D	12. D	13. B	14. C	15. C	16. A
17. C	18. D	19. C	20. C	21. C	22. D	23. B	24. A
25. B	26. B	27. D	28. A	29. C	30. A	31. D	32. D
33. C	34. A	35. C	36. B	37. D	38. A	39. B	40. D
41. B	42. A	43. B	44. C	45. C	46. D	47. D	48. D
49. B	50. A	51. C	52. B	53. A	54. C	55. A	56. A
57. C	58. B	59. B	60. C	61. C	62. B	63. B	64. D
65. B	66. C	67. C					

Explanatory Notes

46. $\cos\angle VCM = \dfrac{CM}{VC}=\dfrac{5}{10}$, $\therefore \angle VCM = 60°$

47. Let N be the mid-point of BC. $MN = 8 \div 2 = 4$.

$\tan\angle VNM = \dfrac{VM}{MN}=\dfrac{5\sqrt{3}}{4}$, $\therefore \angle VNM = 65.2°$

48. Let K be the mid-point of AB. $MK = 6 \div 2 = 3$.

$\tan\angle VKM = \dfrac{VM}{MK}=\dfrac{5\sqrt{3}}{3}$, $\therefore \angle VKM = 70.9°$

59. $\because$ $DB = DH = BH =$ diagonal, $\therefore$ $\angle DHB = 60°$

60. Let the side of cube $= x$, then $EG = \sqrt{x^2 + x^2} = \sqrt{2}x$.

$\tan\theta = \dfrac{\sqrt{2}x}{x} = \sqrt{2}$, $\therefore$ $\theta = 54.7°$

61. Let the side of cube $= x$. $EG^2 = x^2 + x^2 = 2x^2$,

$DG = \sqrt{DE^2 + EG^2} = \sqrt{x^2 + 2x^2} = \sqrt{3}x$, $\therefore$ $MG = \dfrac{\sqrt{3}x}{2}$.

$\sin\dfrac{\theta}{2} = \dfrac{x}{2} \div \dfrac{\sqrt{3}x}{2} = \dfrac{1}{\sqrt{3}}$, $\dfrac{\theta}{2} = 35.26°$, $\therefore$ $\theta = 70.5°$

62. $DF = \sqrt{30^2 + 40^2} = 50$, $\therefore$ $AF = 50\tan 30° = 28.9$ cm

63. $\tan\angle ACF = \dfrac{50\tan 30°}{30}$, $\therefore$ $\angle ACF = 43.9°$

65. $AB = CD = 40$, $BD = AC = \sqrt{(50\tan 30°)^2 + 30^2} = 41.63$;

$\tan\angle ADB = \dfrac{40}{41.63}$, $\therefore$ $\angle ADB = 43.9°$

66. $PN = \sqrt{12^2 + 12^2} \div 2 = 6\sqrt{2}$; $\tan\angle PVN = \dfrac{6\sqrt{2}}{10}$, $\angle PVN = 40.3°$,

$\therefore$ $\angle PVR = 40.3° \times 2 = 80.6°$

67. Let H and K be mid-points of PQ and RS respectively.

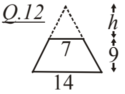
$\tan\angle HVN = \dfrac{HN}{VN} = \dfrac{6}{10}$, $\angle HVN = 30.96°$,

$\therefore$ $\angle HVK = 30.96° \times 2 = 61.9°$

UNIT 11 AREA AND VOLUME (3)

1. D	2. A	3. A	4. B	5. A	6. B	7. B	8. B
9. C	10. A	11. C	12. D	13. B	14. C	15. D	16. C
17. A	18. B	19. C	20. B	21. B	22. D	23. D	24. D
25. C	26. B	27. C	28. B	29. D	30. C	31. C	32. D
33. B	34. A	35. A	36. B	37. D	38. C	39. C	40. A
41. C	42. D	43. A	44. C	45. D	46. B	47. B	48. C
49. C	50. A	51. C	52. A	53. C	54. D	55. A	56. A
57. D	58. C	59. D	60. C	61. A	62. D	63. B	64. A
65. D	66. B	67. D	68. C	69. D	70. A	71. B	

Explanatory Notes

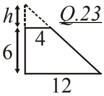


12. $\dfrac{h}{h+9}=\dfrac{7}{14}=\dfrac{1}{2}$, $2h = h + 9$, $h = 9$;

$\therefore$ Volume = $\dfrac{1}{3}(14)^2(9+9)-\dfrac{1}{3}(7)^2(9)=1029\text{ cm}^3$

22. $\pi(3)^2+\pi(3)(\ell) = 33\pi$, $9 + 3\ell = 33$, $\ell = 8$;

$\sin\dfrac{\theta}{2}=\dfrac{3}{8}$, $\dfrac{\theta}{2} = 22.0°$, $\therefore \theta = 44.0°$



23. $\dfrac{h}{h+6}=\dfrac{4}{12}=\dfrac{1}{3}$, $3h = h + 6$, $h = 3$;

$\therefore$ Volume = $\dfrac{1}{3}\pi(12)^2(3+6)-\dfrac{1}{3}\pi(4)^2(3)=416\pi\text{ cm}^3$

24. Curved surface area = $\pi(12)\sqrt{12^2+9^2}-\pi(4)\sqrt{4^2+3^2}$

$= 180\pi - 20\pi = 160\pi\text{ cm}^2$

28. Radius of largest sphere = $8 \div 2$ = 4 cm;

$\therefore$ Volume = $\dfrac{4}{3}\pi(4)^3=268.1\text{ cm}^3$

29. $\dfrac{4}{3}\pi r^3\times 2=\dfrac{4}{3}\pi(10)^3$, $r^3=500$, $\therefore\ r=\sqrt[3]{500}$ = 7.94 cm

30. Percentage change = $\dfrac{4\pi(7.94)^2(2)-4\pi(10)^2}{4\pi(10)^2}\times 100\%$ = 26.1%

34. Let h be height of cylinder.

$\pi(\dfrac{r}{2})^2h=\dfrac{4}{3}\pi r^3$, $(\dfrac{r^2}{4})(h)=\dfrac{4}{3}r^3$, $\therefore\ h=\dfrac{16}{3}r$

36. Let h cm be depth. $\pi(1)^2(h)+\dfrac{2}{3}\pi(1)^3=8\pi$, $h+\dfrac{2}{3}$ = 8, $\therefore h=\dfrac{22}{3}$

42. Let $A\ \text{cm}^2$ be curved surface area.

$\dfrac{y}{A}=\left[\dfrac{r}{r(1+200\%)}\right]^2=\dfrac{1}{9}$, $\therefore A = 9y$

44. $V_A:(V_A+V_B):(V_A+V_B+V_C)=y^3:(y+2y)^3:(y+2y+y)^3$

$= 1 : 27 : 64$, $\therefore\ V_A:V_C = 1 : (64 - 27) = 1 : 37$

45. $S_A:(S_A+S_B):(S_A+S_B+S_C)=y^2:(y+2y)^2(y+2y+y)^2$

$= 1 : 9 : 16$,

$\therefore S_B:S_C = (9 - 1) : (16 - 9) = 8 : 7$

46. $\dfrac{A_1}{A_2}=\left(\dfrac{1}{1-20\%}\right)^2=\dfrac{1}{0.64}$,

$\therefore$ percentage decrease = $(1-0.64)\times 100\%$ = 36%

47. $\dfrac{r_1}{r_2}=\sqrt[3]{\dfrac{1}{1+72.8\%}}=\dfrac{1}{1.2}$,

$\therefore$ percentage change = $(1.2-1)\times 100\%$ = 20%

48. Suppose x cm^3 of water must be added.

$\frac{15}{x+15}=(\frac{1}{2})^3=\frac{1}{8}$, $120 = x + 15$, $\therefore x = 105$

49. $\frac{A_1}{A_2}=(\frac{1}{2})^2=\frac{1}{4}$, $\therefore$ percentage increase = $(4-1)\times 100\%$ = 300%

50. Original volume $V=\frac{1}{3}\pi r^2 h$,

new volume = $\frac{1}{3}[x(1-20\%)]^2[h(1+50\%)]$ = $0.96(\frac{1}{3}x^2 h)=0.96V$,

$\therefore$ percentage change = $\frac{0.96V-V}{V}\times 100\%$ = -4%

51. Ratio = $\frac{1}{3}(\frac{ab}{2})(c):[abc-\frac{1}{3}(\frac{ab}{2})(c)]$ = $\frac{abc}{6}:\frac{5abc}{6}$ = 1 : 5

52. $AB = FG = x$, $GH = y$, $BG = AF = z$;

Vol. of $AEFGH$: vol. of $ABCHG$ = $\frac{1}{3}(\frac{xy}{2})(z):\frac{1}{3}(\frac{yz}{z})(x)$ = 1 : 1

53. $\because \angle AVB = 60° - 30° = 30°$, $\therefore VB = 8$, $VN = 8\sin 60° = 4\sqrt{3}$;

$\therefore$ Volume = $\frac{1}{3}(6\times 8)(4\sqrt{3})$ = 110.9 cm^3

54. Original volume $V=\frac{1}{3}\pi r^2 h$,

new volume = $\frac{1}{3}\pi[r(1+40\%)]^2[h(1-25\%)]$ = $1.47(\frac{1}{3}\pi r^2 h)$ = $1.47V$,

$\therefore$ percentage change = $\frac{1.47V-V}{V}\times 100\%$ = 47%

55. Curved surface area = $\pi(\frac{r}{2})(2\ell)=\pi r\ell$ (unchanged)

56. Height = $12\cos 60°$ = 6 cm, radius = $12\sin 60°\div 2$ = $3\sqrt{3}$ cm,

$\therefore$ volume = $\frac{1}{3}\pi(3\sqrt{3})^2(6)$ = 54π cm^3

57. By cutting along the slant edge through P and flatting the cone to form a sector, the shortest distance is PP'.

$2\pi(15)\times\frac{\theta}{360°}=2\pi(5)$, $\theta = 120°$;

$\therefore PP'$ = $(15\sin\frac{120°}{2})\times 2$ = 26 cm

58. Increase in total surface area = $\pi r^2\times 2$ = $2\pi r^2$,

$\therefore$ percentage change = $\frac{2\pi r^2}{4\pi r^2}\times 100\%$ = 50%

59. I. $= \dfrac{4\pi r^2}{2\pi r(2r)} = \dfrac{4\pi r^2}{4\pi r^2} = 1$

II. $= \dfrac{4\pi r^2}{2\pi r(2r) + 2\pi r^2} = \dfrac{4\pi r^2}{6\pi r^2} = \dfrac{2}{3}$

III. $= \dfrac{\frac{4}{3}\pi r^3}{\pi r^2(2r)} = \dfrac{\frac{4}{3}\pi r^3}{2\pi r^3} = \dfrac{2}{3}$

∴ The answer is D.

60. $\dfrac{V_1}{V_2} = \left(\sqrt{\dfrac{1}{1+125\%}}\right)^3 = (\dfrac{1}{1.5})^3 = \dfrac{1}{3.375}$,

∴ percentage change = $(3.375-1)\times 100\%$ $= 237.5\%$

61. $\dfrac{r_A}{r_B} = \sqrt[3]{\dfrac{27}{125}} = \dfrac{3}{5}$, $\dfrac{r_B}{r_C} = \dfrac{1}{4\div 2} = \dfrac{1}{2}$, ∴ $r_A : r_B : r_C$ $= 3 : 5 :10$,

∴ $(\dfrac{r_A}{r_B})^2 = (\dfrac{3}{10})^2 = \dfrac{9}{100}$. ∵ C is a hemisphere, ∴ $\dfrac{S_A}{S_C} = \dfrac{9}{50}$

62. Let V_W = volume of water, V_E = volume of empty part.

$\dfrac{V_E}{V_E + V_W} = (\dfrac{15-10}{15})^3 = (\dfrac{5}{15})^3 = \dfrac{1}{27}$, ∴ $V_E : V_W$ $= 1 : (27 – 1) = 1 : 26$.

Let d cm be the depth. $\dfrac{d}{15} = \sqrt[3]{\dfrac{26}{27}} = 0.987$, ∴ $d = 14.8$

67. Let $ON = OM = r$. ∵ $\Delta DNC \sim \Delta BMC$,

∴ $\dfrac{ON}{BM} = \dfrac{OC}{BC}$, $\dfrac{r}{6} = \dfrac{8-r}{\sqrt{6^2+8^2}}$, $10r = 48 – 6r$, $16r = 48$, ∴ $r = 3$

69. Let d cm be the depth. $\dfrac{d}{8} = \sqrt[3]{\dfrac{3}{8}} = 0.721$, ∴ $d = 5.77$

70. $\dfrac{1}{3}\pi r^2 h = \dfrac{2}{3}\pi r^3(1-25\%)$, $h = 2r(0.75)$, $\dfrac{h}{r} = 1.5 = \dfrac{3}{2}$, ∴ $h : r = 3 : 2$

71. $h = 30\times\dfrac{3}{3+2} = 18$, $r = 30 – 18 = 12$,

∴ volume = $\dfrac{1}{3}\pi(12)^2(18) + \dfrac{2}{3}\pi(12)^3 = 2016\pi \text{ cm}^3$

UNIT 12 COORDINATES OF STRAIGHT LINES

1. A	2. C	3. C	4. D	5. A	6. D	7. D	8. B
9. A	10. B	11. C	12. C	13. C	14. A	15. B	16. A
17. D	18. B	19. D	20. A	21. C	22. D	23. D	24. C
25. A	26. C	27. B	28. A	29. D	30. D	31. B	32. C
33. A	34. D	35. B	36. A	37. A	38. B	39. C	40. B
41. A	42. D	43. C	44. D	45. A	46. D	47. B	48. C
49. A	50. C	51. C	52. B	53. D	54. D	55. C	56. B
57. B	58. C	59. D	60. B	61. B	62. B	63. D	64. A
65. B	66. A	67. A	68. B	69. A	70. C	71. D	72. A

Explanatory Notes

6. I. $AB=\sqrt{10}$, $BC=2\sqrt{5}$, $AC=\sqrt{10}$, $\therefore$ ΔABC is isosceles.

II. $AB^2+AC^2=(\sqrt{10})^2+(\sqrt{10})^2=20=BC^2$,

$\therefore$ ΔABC is rt.$\angle$ed.

III. Area $=\dfrac{\sqrt{10}\times\sqrt{10}}{2}$ = 5 sq. units

$\therefore$ The answer is D.

38. Let y-intercept = a. $\dfrac{a-0}{0-(-10)}\times 1.25=-1$, $\dfrac{a}{10}\times\dfrac{5}{4}=-1$, $\therefore$ $a=-8$

40. $m_{PQ}=\dfrac{3-1}{3+1}=\dfrac{1}{2}$, $m_{QR}=\dfrac{3-1}{3-4}=-2$, $m_{RS}=\dfrac{1+1}{4-0}=\dfrac{1}{2}$,

$m_{PS}=\dfrac{1+1}{-1-0}=-2$.

$\because$ $m_{PQ}=m_{RS}$ and $m_{QR}=m_{PS}$, $\therefore$ PQ // RS and QR // PS;

$\because$ $m_{PQ}\times m_{QR}=m_{RS}\times m_{PS}=(\frac{1}{2})(-2)=-1$, $\therefore$ $PQ\perp QR$ and $RS\perp PS$;

But $PQ=\sqrt{(3+1)^2+(3-1)^2}=2\sqrt{5}$, $QR=\sqrt{(4-3)^2+(1-3)^2}=\sqrt{5}$,

$\because$ $PQ\neq QR$, $\therefore$ $PQRS$ is a rectangle.

48. $3QR=PQ=PR+QR$, $2QR=PR$, $\therefore$ $PR:QR=2:1$,

$\therefore$ $R=\left(\dfrac{1(-10)+2(2)}{1+2},\dfrac{1-(-1)+2(5)}{1+2}\right)=(-2, 3)$

49. Let $B=(x, y)$. $\dfrac{9(3)+x(1)}{1+3}=3$, $27+x=12$, $x=-15$;

$\dfrac{-4(3)+y(1)}{1+3}=2$, $-12+y=8$, $y=20$. $\therefore$ $B=(-15, 20)$

50. $AC:BC=[(-3)-(-6)]:[4.5-(-3)]=3:7.5=2:5$

51. $\because$ x-coordinate of $P = 0$,
$\therefore$ $AP : PB = (6 - 0) : [0 - (-10)] = 6 : 10 = 3 : 5$

52. $PR : QR = [(k + 4) - k] : [k - (k - 1)] = 4 : 1$

53. $\sqrt{(-5+3)^2+(k-7)^2} = 2\sqrt{5}$, $4+(k-7)^2 = 20$, $(k-7)^2 = 16$,
$k - 7 = -4$ or 4, $\therefore$ $k = 3$ or 11

55. $\theta = \tan^{-1}(2) - \tan^{-1}(\frac{1}{2}) = 63.43° - 26.57° = 36.9°$ (ext. $\angle$ of Δ)

56. Slope $= \tan\theta = \dfrac{\sqrt{13^2 - 12^2}}{12} = \dfrac{5}{12}$

58. Let $B = (0, y)$. $\because$ $L_1 \perp L_2$, $\therefore$ $\dfrac{y-1}{0-8} \times \dfrac{5-1}{0-8} = -1$, $\dfrac{y-1}{-8} \times \dfrac{1}{-2} = -1$,
$y - 1 = -16$, $y = -15$. $\therefore$ Area $= \dfrac{1}{2}(5+15)(8) = 80$ sq. units

59. Let y-intercept of $L_1 = k$, then x-intercept of $L_1 = 2k$. $\because$ $L_1 \perp L_2$,
$\therefore$ $\dfrac{k-0}{0-2k} \times \dfrac{b-0}{a-0} = -1$, $\dfrac{k}{-2k} \times \dfrac{b}{a} = -1$, $\dfrac{b}{a} = 2$, $\therefore$ $b = 2a$

60. Mid-point (M) of $PR = \left(\dfrac{3+1}{2}, \dfrac{6-4}{2}\right) = (2, 1)$.
Let $S = (x, y)$. $\because$ M is also the mid-pt. of QS (prop. of // gram),
$\therefore$ $\dfrac{x-2}{2} = 2$, $x = 6$; $\dfrac{y+2}{2} = 1$, $y = 0$. $\therefore$ $S = (6, 0)$

61. Let $A = (x, 0)$, $B = (0, y)$. $\dfrac{x(2)+0(1)}{1+2} = 3$, $2x = 9$, $x = 4.5$;
$\dfrac{y(1)+0(2)}{1+2} = 5$, $y = 15$. $\therefore$ $A = (4.5, 0)$, $B = (0, 15)$

62. Let D = $(x, 0)$. Mid-point of AB = $\left(\dfrac{-6+0}{2}, \dfrac{0+12}{2}\right) = (-3, 6)$.
$\because$ AB$\perp$CD, $\therefore$ $\dfrac{12-0}{0+6} \times \dfrac{0-6}{x+3} = -1$, $\dfrac{-12}{x+3} = -1$, $x + 3 = 12$, $x = 9$.
$\therefore$ D = $(9, 0)$

63. Let B = $(x, 0)$. $\because$ A, B, D are collinear, $\therefore$ $\dfrac{0-6}{x-16} = \dfrac{6+9}{16+4} = \dfrac{3}{4}$,
$-24 = 3x - 48$, $x = 8$; $\because$ A, C, D are collinear, $\therefore$ $\dfrac{y-6}{0-16} = \dfrac{3}{4}$,
$4y - 24 = -48$, $y = -6$. $\therefore$ Area $= \dfrac{1}{2}(8)(6) = 24$ sq. units

65. $\angle AOX = \tan^{-1}(\frac{6}{3}) = 63.43°$, $\angle COX = \tan^{-1}(\frac{2}{4}) = 26.57°$,
$\therefore$ $\angle BOX = 26.57° + (63.43° - 26.57°) \div 2 = 45°$,
$\therefore$ slope $= \tan 45° = 1$

66. $\because AM = MB$ and $AN = NC$, $\therefore MN = \frac{1}{2}BC$ (mid-pt. thm.),

$\therefore MN = \frac{1}{2}\sqrt{(-6-10)^2 + (7+5)^2} = \frac{1}{2}(20) = 10$

67. $\because \Delta AOC$ and ΔBOC have the same height,

$\therefore AC : CB =$ area of ΔAOC : area of $\Delta BOC = 2 : 3$,

$\therefore C = \left(\frac{3(-8)+2(0)}{2+3}, \frac{3(0)+2(-5)}{2+3}\right) = (-4.8, -2)$

68. $M = \left(\frac{3+20}{2}, \frac{0+2}{2}\right) = (11.5, 1)$,

$\therefore G = \left(\frac{1(10)+2(11.5)}{1+2}, \frac{1(10)+2(1)}{1+2}\right) = (11, 4)$

69. Circumcentre $= \left(\frac{18+0}{2}, \frac{0+24}{2}\right) = (9, 12)$

70. Radius $= \frac{\sqrt{(0-18)^2 + (24+0)^2}}{2} = 15$,

$\therefore$ area $= \pi(15)^2 = 225\pi$ sq. units

71. $P = \left(\frac{-2+8}{2}, \frac{3+5}{2}\right) = (3, 4)$, $Q = \left(\frac{-2+0}{2}, \frac{3-3}{2}\right) = (-1, 0)$.

Let $C = (x, y)$. $\because PC \perp XY$, $\therefore \frac{y-4}{x-3} \times \frac{5-3}{8+2} = -1$, $\frac{y-4}{x-3} = -5$,

$y - 4 = -5x + 15$, $5x + y = 19$ ……(1); $\because CQ \perp XZ$,

$\therefore \frac{y-0}{x+1} \times \frac{3+3}{-2-0} = -1$, $\frac{y}{x+1} = \frac{1}{3}$, $3y = x + 1$,

$x - 3y = -1$ ……(2); Solving (1) and (2), we have $x = 3.5$, $y = 1.5$. $\therefore C = (3.5, 1.5)$

72. Let $H = (x, y)$. $\because PH \perp RQ$, $\therefore \frac{y-5}{x-5} \times \frac{0+1}{-3-5} = -1$, $\frac{y-5}{x-2} = 8$,

$y - 5 = 8x - 16$, $8x - y = 11$ ……(1); $\because QH \perp PR$,

$\therefore \frac{y+1}{x-5} \times \frac{5-0}{2+3} = -1$, $\frac{y+1}{x-5} = -1$, $y + 1 = -x + 5$,

$x + y = 4$ ……(2); Solving (1) and (2), we have $x = \frac{5}{3}, y = \frac{7}{3}$.

$\therefore H = \left(\frac{5}{3}, \frac{7}{3}\right)$

UNIT 13 TRIGONOMETRIC RELATIONS

1. A	2. D	3. D	4. C	5. B	6. A	7. C	8. A
9. D	10. D	11. B	12. C	13. B	14. A	15. B	16. C
17. A	18. C	19. A	20. A	21. C	22. D	23. C	24. B
25. D	26. A	27. D	28. B	29. A	30. C	31. B	32. A
33. B	34. B	35. D	36. A	37. C	38. C	39. D	40. C
41. D	42. D	43. B	44. A	45. B	46. B	47. B	48. C
49. A	50. D	51. A	52. C	53. C	54. D	55. A	56. C
57. A	58. B	59. B	60. A	61. D	62. C	63. C	64. D
65. B	66. A	67. C	68. A				

Explanatory Notes

17. $AB = 4 \div \tan 45° = 4$, $BD = 4 \div \sin 45 = 4\sqrt{2}$,
$CD = 4\sqrt{2}\sin 30° = 2\sqrt{2}$, $BC = 4\sqrt{2}\cos 30° = 2\sqrt{6}$,
$\therefore$ area $= \dfrac{4\times 4}{2} + \dfrac{2\sqrt{2}\times 2\sqrt{6}}{2} = (4\sqrt{3}+8)\text{ cm}^2$

22. $\because AS : AP : PS = 1 : \sqrt{3} : 2$, $\therefore AB : PS = (1+\sqrt{3}) : 2$,
$\therefore$ area of $ABCD$: area of $PQRS = (1+\sqrt{3})^2 : 2^2 = (4+2\sqrt{3}) : 4$
$= (2+\sqrt{3}) : 2$

23. $\because$ X, Y and Z are similar, $\therefore X : Y : Z = 1^2 : (\sqrt{3})^2 : 2^2 = 1 : 3 : 4$

24. $\sqrt{12} - \sqrt{6}\cos(x+5°) = \sqrt{3}$, $2\sqrt{3} - \sqrt{3} = \sqrt{6}\cos(x+5°)$,
$\cos(x+5°) = \dfrac{\sqrt{3}}{\sqrt{6}} = \dfrac{1}{\sqrt{2}}$, $x + 5° = 45$, $\therefore x = 40°$

25. $1+\tan x = \dfrac{2}{\sqrt{3}-1} = \dfrac{2(\sqrt{3}+1)}{3-1} = \sqrt{3}+1$, $\tan x = \sqrt{3}$, $\therefore x = 60°$

30. $= \dfrac{(1-\cos x)-(1+\cos x)}{1^2-\cos^2 x} = -\dfrac{2\cos x}{\sin^2 x}$

31. $= \dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

32. $= \dfrac{\sin^4 x + \sin^2 x\cos^2 x}{\cos^2 x} = \dfrac{\sin^2 x(\sin^2 x+\cos^2 x)}{\cos^2 x} = \tan^2 x$

33. $= (\dfrac{\sin^2\theta - 1}{\sin\theta})(\dfrac{\cos^2\theta - 1}{\cos\theta}) = (\dfrac{-\cos^2\theta}{\sin\theta})(\dfrac{-\sin^2\theta}{\cos\theta}) = \sin\theta\cos\theta$

35. $5\sin^2\theta + 4\cos^2\theta = 5$, $5\sin^2\theta + 4(1-\sin^2\theta) = 5$, $\sin^2\theta = 1$,
$\therefore \sin\theta = 1$

38. $= \tan\theta + \dfrac{1}{\tan\theta} = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\sin\theta\cos\theta}$

39. $= \dfrac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x + \sin^2 x = 2\sin^2 x$

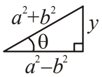
40. $= \cos\theta + \sin\theta \cdot \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \dfrac{1}{\cos\theta}$

43. $\because y^2 = (a^2+b^2)^2 - (a^2-b^2)^2$

$= (a^2+b^2+a^2-b^2)(a^2+b^2-a^2+b^2)$

$= 4a^2b^2$, $\therefore$ $y = 2ab$, $\therefore$ $\tan\theta = \dfrac{2ab}{a^2-b^2}$

Q.43



44.
$$\begin{array}{rl} 10x &= 4.444\ldots\ldots \\ -\quad x &= 0.444\ldots\ldots \\ \hline 9x &= 4 \end{array}$$
$\therefore x = \tan\theta = \dfrac{4}{9}$,

$\therefore$ $\sin\theta - \cos\theta = \dfrac{4}{\sqrt{4^2+9^2}} - \dfrac{9}{\sqrt{4^2+9^2}} = \dfrac{-5}{\sqrt{97}} = \dfrac{-5\sqrt{97}}{97}$

45. $\sqrt{3}\sin 2\theta = \dfrac{3}{2}$, $\sin 2\theta = \dfrac{\sqrt{3}}{2}$, $2\theta = 60°$, $\therefore \theta = 30°$

46. $\cos\theta - \sqrt{3}\sin\theta = 0$, $\cos\theta = \sqrt{3}\sin\theta$, $\tan\theta = \dfrac{1}{\sqrt{3}}$, $\therefore$ $\theta = 30°$

48. $2\sin(x+y) = \sqrt{3}$, $\sin(x+y) = \dfrac{\sqrt{3}}{2}$, $x+y = 60°$ …… (1);

$3\tan(x-y) = \sqrt{3}$, $\tan(x-y) = \dfrac{\sqrt{3}}{3}$, $x-y = 30°$ …… (2);

Solving (1) and (2), we have $x = 45°, y = 15°$.

49. $x\tan 60° - \sin 30° \le x\tan 45° + \cos 30°$, $x(\sqrt{3}) - \dfrac{1}{2} \le x + \dfrac{\sqrt{3}}{2}$,

$x(\sqrt{3}-1) \le \dfrac{\sqrt{3}+1}{2}$, $x \le \dfrac{\sqrt{3}+1}{2(\sqrt{3}-1)}$, $x \le \dfrac{(\sqrt{3}+1)^2}{2(3-1)}$, $x \le \dfrac{4+2\sqrt{3}}{4}$,

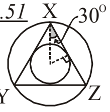
$\therefore$ $x \le \dfrac{2+\sqrt{3}}{2}$

50. $\because AB = PR =$ diameter of circle, $\therefore AB : PQ = PR : PQ = \sqrt{2} : 1$

51. $\because$ Radius of C_1 : radius of C_2 $= 2 : 1$,

$\therefore$ area of C_1 : area of C_2 $= 2^2 : 1^2 = 4 : 1$

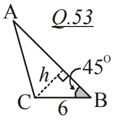
Q.51



53. $h = 6\sin 45° = 6(\dfrac{1}{\sqrt{2}}) = 3\sqrt{2}$;

$\dfrac{AB \times 3\sqrt{2}}{2} = 27$, $\therefore$ $AB = \dfrac{54}{3\sqrt{2}} = 9\sqrt{2}$

Q.53



54. Let $AB = BC = a$.

$CD = \dfrac{2a}{\tan 60°} = \dfrac{2a}{\sqrt{3}} = \dfrac{2\sqrt{3}a}{3}$, $CE = \dfrac{a}{\tan 30°} = \sqrt{3}a$,

$\therefore CD : DE = \frac{2\sqrt{3}a}{3} : (\sqrt{3}a - \frac{2\sqrt{3}a}{3}) = \frac{2\sqrt{3}a}{3} : \frac{\sqrt{3}a}{3} = 2:1$

55. Let $AD = DC = a$.

$BC = \frac{2a}{\cos 30°} = 2a \times \frac{2}{\sqrt{3}} = \frac{4\sqrt{3}a}{3}$, $EC = a\cos 30° = \frac{\sqrt{3}a}{2}$,

$\therefore BE : EC = (\frac{4\sqrt{3}a}{3} - \frac{\sqrt{3}a}{2}) : \frac{\sqrt{3}a}{2} = \frac{5\sqrt{3}a}{6} : \frac{\sqrt{3}a}{2} = 5 : 3$

56. Let $AD = BD = a$.

$\angle ABD = 30°$ (base ∠s, isos. Δ), $\angle BDC = 60°$ (ext. ∠ of Δ),

$\therefore CD = BD \cos 60° = \frac{a}{2}$, $\therefore CD : AD = \frac{a}{2} : a = 1 : 2$

57. $= (\sin^2 x + \cos^2 x)^2 = 1$

58. $= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = (1)(1 - \sin^2 x - \sin^2 x)$

$= 1 - 2\sin^2 x$

59. $= \sin^2\theta + \cos^2\theta\,(\sin^2\theta + \cos^2\theta) = \sin^2\theta + \cos^2\theta(1) = 1$

60. $= \frac{\cos\theta\,(1+\cos\theta)}{1+\cos\theta} + \frac{\sin^2\theta}{1+\cos\theta} = \frac{\cos\theta + \cos^2\theta + \sin^2\theta}{1+\cos\theta}$

$= \frac{\cos\theta + 1}{1+\cos\theta} = 1$

61. $= \cos^2\theta\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) = \cos^2\theta\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right) = \sin\theta\cos\theta$

62. $= \frac{\cos\theta}{1-\sin\theta} + \frac{1-\sin\theta}{\cos\theta} = \frac{\cos^2\theta + (1-\sin\theta)^2}{\cos\theta(1-\sin\theta)}$

$= \frac{\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta}{\cos\theta\,(1-\sin\theta)} = \frac{2 - 2\sin\theta}{\cos\theta\,(1-\sin\theta)} = \frac{2}{\cos\theta}$

65. $\sin\theta + \cos\theta = \frac{3}{2}$, $(\sin\theta + \cos\theta)^2 = (\frac{3}{2})^2$,

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = \frac{9}{4}$, $2\sin\theta\cos\theta = \frac{9}{4} - 1$,

$\therefore \sin\theta\cos\theta = \frac{5}{4} \times \frac{1}{2} = \frac{5}{8}$

66. $= \sin^2 1° + \sin^2 2° + \ldots\ldots + \sin^2 44° + \sin^2 45° + \cos^2 44° + \ldots\ldots$
$+ \cos^2 2° + \cos^2 1°$

$= (\sin^2 1° + \cos^2 1°) + \ldots\ldots + (\sin^2 44° + \cos^2 44°) + \sin^2 45°$

$= 44 \times 1 + (\frac{1}{\sqrt{2}})^2 = 44.5$

67. $= \tan 2° \times \tan 4° \times \ldots \times \tan 44° \times \frac{1}{\tan 44°} \times \ldots\ldots \times \frac{1}{\tan 4°} \times \frac{1}{\tan 2°} = 1$

68. $\tan\theta\tan(\theta+20°)=1$, $\frac{1}{\tan(90°-\theta)}\times\tan(\theta+20°)=1$,

$\tan(\theta+20°)=\tan(90°-\theta)$, $\theta+20°=90°-\theta$, $2\theta=70°$,

$\therefore\ \theta=35°$

UNIT 14 APPLICATION OF TRIGONOMETRY

1. B	2. B	3. D	4. C	5. A	6. C	7. B	8. B
9. B	10. D	11. D	12. C	13. C	14. A	15. C	16. A
17. B	18. A	19. D	20. A	21. B	22. B	23. A	24. A
25. D	26. C	27. D	28.C	29. D	30. A	31. B	32. C
33. B	34. D	35. D	36. C	37. A	38. B	39. A	40. C
41. D	42. A	43. B	44. A	45. B	46. C	47. B	48. C
49. D	50. C	51. B	52. B	53. A	54. A	55. C	56. D
57. D	58. C	59. B	60. B	61. D	62. C	63. B	64. B
65. D	66. C	67. B	68. D	69. A	70. A	71. D	72. A
73. C	74. A	75. C	76. B				

Explanatory Notes

14. Let θ be the inclination of second slope.

$\because\ \tan\theta=\frac{1}{3}$, $\therefore\ \sin\theta=\frac{1}{\sqrt{1^2+3^2}}=\frac{1}{\sqrt{10}}$, $\cos\theta=\frac{3}{\sqrt{10}}$.

Total vertical distance = $220\sin 14°+160\times\frac{1}{\sqrt{10}}$ = 103.82,

total horizontal distance = $220\cos 14°+160\times\frac{3}{\sqrt{10}}$ = 365.25,

$\therefore$ angle of depression = $\tan^{-1}(\frac{103.82}{365.25})$ = 15.9°

16. Angle of elevation = $\tan^{-1}(\frac{4}{12})$ = 18.4°

20. Let x m be the height of the flagstaff.

$\frac{x}{\tan 46°}+\frac{x}{\tan 25°}=150$, $x(\frac{1}{\tan 46°}+\frac{1}{\tan 25°})=150$,

$\therefore\ x=150\div(\frac{1}{\tan 46°}+\frac{1}{\tan 25°})=48.2$

21. $\frac{OR}{\tan 20°}-\frac{OR}{\tan 65°}=10\times 15$, $OR(\frac{1}{\tan 20°}-\frac{1}{\tan 65°})=150$,

$\therefore\ OR=150\div(\frac{1}{\tan 20°}-\frac{1}{\tan 65°})=65.8$ m

22. Let h m be the height.

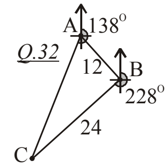
$\frac{h}{\tan 72°}=\frac{h-55}{\tan 39°}$, $h\tan39° = h\tan72° - 55\tan72°$,

$h(\tan72° - \tan 39°) = 55\tan72°$, $\therefore h = 74.6$

23. $h+\frac{h}{\tan 24°}\times\tan 35°=120$, $h(1+\frac{\tan 35°}{\tan 24°})=120$, $\therefore h = 46.6$

32. $\angle ABC = 360° - 228° - (180° - 138°) = 90°$,

$\therefore AC=\sqrt{12^2+24^2}=\sqrt{720}=12\sqrt{5}$ km



34. $\angle PAB = 180° - 156° = 24°$,

$\angle PBA = 270° - 225° = 45°$.

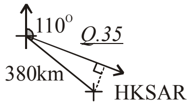
Let x m be the shortest distance.

$\frac{x}{\tan 24°}+\frac{x}{\tan 45°}=460$, $x(\frac{1}{\tan 24°}+1)=460$,

$\therefore x=460\div(\frac{1}{\tan 24°}+1)=142$

35. Shortest distance = $380\sin(180° - 110° - 45°)$

= $380\sin25°$ = 160.6 km



36. Time taken = $380\cos 25°\div 100$ = 3.4 h

43. The pentagon is formed by five identical isosceles triangles.

Each base angle = $(5-2)\times 180°\div 5\div 2$ = 54°,

base = $15\cos 54°\times 2$ = $30\cos54°$, height = $15\sin54°$,

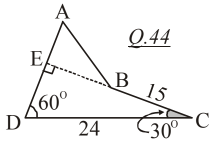
$\therefore$ area = $\pi(15)^2-\frac{30\cos 54°\times 15\sin 54°}{2}\times 5$ = 172 cm^2

44. $DE = 24\cos60° = 12$, $CE = 24\sin60° = 12\sqrt{3}$,

$AE = 16 - 12 = 4$, $BE = 12\sqrt{3}-15$,

$\therefore$ area = $\frac{12\times 12\sqrt{3}}{2}+\frac{4(12\sqrt{3}-15)}{2}$

= 136.3 cm^2



45. $\sin\theta=\frac{2\sin 45°}{7}=\frac{\sqrt{2}}{7}$, $\therefore \theta=11.7°$

46. Height = $4\sin(180° - 90° - 11.7°) = 4\sin78.3°$ = 3.9 cm

47. Height = $3.9 + 7\sin11.7°$ = 5.3 cm

48. Let $AB = AD = DE = BE = x$ cm. $\frac{x}{\tan 40°}+x+\frac{x}{\tan 60°}=9$,

$x(\frac{1}{\tan 40°}+1+\frac{1}{\tan 60°})=9$, $x=9\div(\frac{1}{\tan 40°}+1+\frac{1}{\tan 60°})=3.25$.

$\therefore$ Area = $\frac{(3.25+9)(3.25)}{2}$ = 19.9 cm^2

49. Let r cm be the radius. $\frac{r}{\sin 30°}+r=18$, $2r + r = 18$, $\therefore r = 6$

52. Let a be vertical distance between A and B.

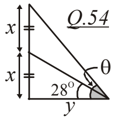
Slope of $AB=\frac{a}{4}$, slope of $CD=\frac{2a}{5}$, slope of $EF=\frac{3a}{8}$,

$\because\ \frac{2a}{5}>\frac{3a}{8}>\frac{a}{4}$, $\therefore$ CD has the greatest gradient.

53. Let a be vertical distance between A and B. $\tan 10°=\frac{a}{4}$, $\therefore\ a=$ 4tan 10°.

Let θ be inclination of PQ. $\tan\theta=\frac{2a}{6}=\frac{4\tan 10°}{3}$, $\therefore\ \theta=13.2°$

54. $\tan 28°=\frac{x}{y}$. Let θ be the angle of depression.



Q.54

$\tan\theta=\frac{2x}{y}=2\tan 28°$, $\therefore\ \theta=46.8°$

59. $\tan\angle OPQ=\frac{30}{60}$, $\angle OPQ=26.57°$;

$\sin\angle OPG=\frac{20}{\sqrt{30^2+60^2}}$, $\angle OPG=17.35°$;

$\therefore$ Angle of elevation = 26.57° + 17.35° = 43.9°

60. $\because\ \angle SBC=20°+40°=60°$ and $\frac{BC}{SB}=\frac{100}{50}=2$, $\therefore\ \angle CSB=90°$,

$\therefore\ SC=100\sin 60°=50\sqrt{3}$ m

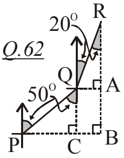
61. $\angle SCB=180°-90°-60°=30°$,

$\therefore$ the bearing is S(30° + 40°)W or S70°W.

62. $RB=RA+QC=8\cos 20°+12\cos 50°=15.23$,

$PB=PC+QA=12\sin 50°+8\sin 20°=11.93$,

$\therefore\ PR=\sqrt{15.23^2+11.93^2}=19.3$ km

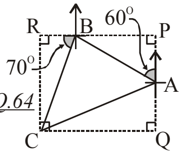


Q.62

64. $AQ=RC-PA=190\sin 70°-140\cos 60°$
$=108.54$,

$CQ=RB+BP=190\cos 70°+140\sin 60°$
$=186.23$,

$\therefore$ distance $=AC=\sqrt{108.54^2+186.23^2}$
$=216$ m

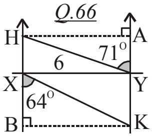


Q.64

66. $AK=AY+BX=\frac{6}{\tan 71°}+\frac{6}{\tan 64°}=4.99$;

$\tan\angle AKH=\frac{6}{4.99}$, $\angle AKH=50.2°$,

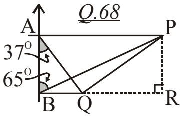
$\therefore$ bearing of H from K is N50.2°W.



Q.66

68. $QR=AP-BQ=80\tan 65°-80\tan 37°$
$=111.28$;

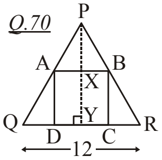
$\tan\angle QPR=\frac{111.28}{80}$, $\angle QPR=54°$,



Q.68

∴ bearing of Q from P is 180° + 54° or 234°.

70. $PY = \frac{12}{2} \times \tan 60° = 6\sqrt{3}$.



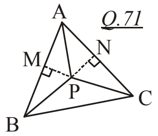
Let a cm be the side of square $ABCD$.

∵ $\Delta PAB \sim \Delta PQR$, ∴ $\frac{AB}{QR} = \frac{PX}{PY}$,

$\frac{a}{12} = \frac{6\sqrt{3} - a}{6\sqrt{3}}$, $6\sqrt{3}a = 72\sqrt{3} - 12a$,

$(6\sqrt{3} + 12)a = 72\sqrt{3}$, ∴ $a = 5.57$

71. $\angle CAP = \angle BAP = 46° \div 2 = 23°$,



$\angle CBP = \angle ABP = 62° \div 2 = 31°$,

$\angle ACP = \angle BCP = (180° - 62° - 46°) \div 2 = 36°$.

$PM = PN = 4\sin 23°$,

∴ $BP = \frac{PM}{\sin 31°} = \frac{4\sin 23°}{\sin 31°} = 3.03$ cm,

$CP = \frac{PN}{\sin 36°} = \frac{4\sin 23°}{\sin 36°} = 2.66$ cm

75. ∵ $\Delta BMX \cong \Delta AMX$, ∴ $\angle MAX = \angle MBX = 50° \div 2 = 25°$,

but $\angle BAC = (180 - 50°) \div 2 = 65°$, ∴ $\angle MAN = 65° - 25° = 40°$,

∴ $MN = AN\tan 40° = \frac{16}{2} \times \tan 40° = 6.71$ cm

76. $AM = \frac{AN}{\cos 40°} = \frac{8}{\cos 40°} = 10.44$,

∴ area = $\pi(10.44)^2 = 342.6 \text{ cm}^2$

UNIT 15 INTRODUCTION TO PROBABILITY

1. D	2. B	3. A	4. C	5. C	6. B	7. C	8. B
9. D	10. C	11. D	12. C	13. A	14. A	15. D	16. B
17. B	18. C	19. A	20. A	21. C	22. C	23. A	24. D
25. C	26. B	27. D	28.D	29. A	30. A	31. B	32. B
33. C	34. A	35. B	36. D	37. C	38. A	39. C	40. B
41. C	42. D	43. C	44. D	45. C	46. B	47. B	48. D
49. A	50. B	51. A	52. A	53. D	54. D	55. C	56. B
57. A	58. D	59. C	60. B	61. A	62. C	63. C	64. B

Explanatory Notes

6. 2, 3 and 5 are prime numbers.

54. $\dfrac{\text{Area of shaded region}}{\text{Area of whole target}} = (\dfrac{a}{3a})^2 = \dfrac{1}{9}$,

$\therefore$ required probability $= \dfrac{9-1}{9} = \dfrac{8}{9}$

55. Let x = total no. of balls. $\dfrac{x-20}{x} = \dfrac{4}{9}$, $9x - 180 = 4x$, $\therefore x = 36$

57. Total no. of balls $= 18 \div (1 - \dfrac{1}{10} - \dfrac{3}{5}) = 18 \div \dfrac{3}{10} = 60$,

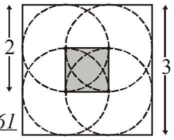
$\therefore$ difference $= 60 \times (\dfrac{3}{10} - \dfrac{1}{10}) = 12$

58. Total no. of coins $= 12 \div (1 - \dfrac{3}{4}) = 48$

59. Expected value $= 1 \times \dfrac{1}{6} + 2 \times \dfrac{1}{6} + 3 \times \dfrac{1}{6} + 4 \times \dfrac{1}{6} + 5 \times \dfrac{1}{6} + 6 \times \dfrac{1}{6} = 3.5$

61. The player can win if the centre of the coin lies on the middle square of side (3 – 1 – 1) = 1 cm.

$\therefore$ Required probability $= \dfrac{1 \times 1}{3 \times 3} = \dfrac{1}{9}$



Q.61

62. Suppose A is fixed, possible outcomes are:

A B D C	A B C D	A C D B	A C B D	A D C B	A D B C

$\therefore$ Required probability $= \dfrac{4}{6} = \dfrac{2}{3}$

63. Possible outcomes: (3, 5, 7), (3, 5, 9), (3, 7, 9), (5, 7, 9)
Favourable outcomes: (3, 5, 7), (3, 7, 9), (5, 7, 9)

$\therefore$ Required probability $= \dfrac{3}{4}$

64. The longer part is at least 30 cm longer than the shorter part when the piece of string is cut within regions AB and CD.

$\therefore$ Required probability $= \dfrac{10+10}{50} = \dfrac{2}{5}$

UNIT 16 MEASURES OF CENTRAL TENDENCY

1. C	2. B	3. B	4. A	5. D	6. D	7. C	8. C
9. D	10. D	11. B	12. A	13. C	14. D	15. A	16. B
17. D	18. C	19. A	20. D	21. D	22. D	23. A	24. A
25. D	26. B	27. C	28.C	29. D	30. B	31. B	32. A
33. D	34. D	35. C	36. D	37. B	38. A	39. A	40. D
41. D	42. D	43. D	44. B	45. C	46. B	47. A	48. B
49. B	50. C	51. C	52. D	53. A	54. B	55. B	56. A
57. D	58. A	59. C	60. A	61. D	62. B	63. C	64. C

Explanatory Notes

6. $8\times15+12n=9.5(15+n)$, $120 + 12n = 142.5 + 9.5n$, $\therefore n = 9$

7. $a + b + c + d = 18\times4 = 72$,

$$\therefore \text{mean} = \frac{(2a+1)+(b-4)+(c-5)+(9-a)+(d+7)}{5}$$

$$= \frac{(a+b+c+d+e)+8}{5} = \frac{72+8}{5} = 16$$

8. $nm - 7 - 12 - 23 = m(n-3)$, $nm - 42 = nm - 3m$, $\therefore m = 14$

9. Original mean $= (28+35+19+44+24)\div5 = 150\div5 = 30.$

Let x be the number.

$\dfrac{150+x}{6} = 30(1+20\%)$, $150+x = 36\times6$, $\therefore x = 66$

14. Present mean age $= \dfrac{(18+6)\times16-27}{15} = 23.8$

15. Let x = no. of men, y = no. of woman. $178x + 158y = 165.5(x + y)$,

$12.5x = 7.5y$, $\dfrac{y}{x} = \dfrac{12.5}{7.5} = \dfrac{5}{3}$, $\therefore x : y = 5 : 3$

18 Rearrange the data: $\dfrac{3k}{5}, \dfrac{2k}{3}, \dfrac{5k}{7}, \dfrac{3k}{4}, \dfrac{5k}{6}$;

$\therefore \dfrac{5k}{7} = 15$, $k = 21$

20. $\because$ The magnitude and sign of x are not known,

$\therefore$ the median cannot be determined.

21. x can be 5, 6, 7, 8.

23. $\because \dfrac{8+10}{2} = 9$, $\therefore a$ should be arranged after 10,

$\therefore a \geq 10$, that means, $a > 9$

25. $\because$ 6 and 7 are smaller than 8 which is the median,

$\therefore$ there are two cases:

(1) $p - 7$ and $p - 2$ are the middle 2 numbers, then

$\frac{(p-7)+(p-2)}{2}=8$, $2p-9$, $p = 12.5$

(2) 7 and $p - 2$ are the middle 2 numbers, then

$\frac{7+(p-2)}{2}=8$, $p + 5 = 16$, $p = 11$

$\because$ p is an integer, $\therefore$ $p = 11$

33. For example, original set of numbers can be –1, –1, 1, 1, x, x, x.
When squared, the set becomes 1, 1, 1, 1, x^2, x^2, x^2.
$\because$ The mode is changed, $\therefore$ the new mode cannot be determined.

35. Mean = $\frac{3^{4x+1}+9^{2x+1}+81^{x+1}}{3} = \frac{3^{4x+1}+3^{4x+2}+3^{4x+4}}{3}$

$= \frac{3^{4x+1}(1+3+3^3)}{3} = 3^{4x}\cdot 31$

40. If the mean, mode and median are negative, they will become larger when multiplied by –3.

43. $\because$ Mode = 15, $\therefore$ a = 15. $13+15+15+b+19+22=17\times 6$,
$\therefore$ b = 102 – 84 = 18

44. $\because$ Median = 10, $\therefore$ c = 10. $\because$ Mode = 8, $\therefore$ a = b = 8.
$8+8+10+d+e=10\times 5$, $d + e = 24$, but d and e should be different integers which are greater than 10, $\therefore$ $d = 11$, $e = 13$

45. $a=18\times 4\times \frac{2}{2+5+2+3}=72\times\frac{1}{6}=12$

46. Let $a = 2k$, $b = 5k$, $c = 2k$, $d = 3k$.
$\frac{2k+3k}{2}=35$, $5k = 70$, $k = 14$, $\therefore$ $d = 3(14) = 42$

47. $2+x+y+17=9\times 4$, $x + y = 17$ ……(1);
$2\times 5+3x+6y+17\times 6=9.8(5+3+6+6)$, $3x + 6y = 84$ ……(2);
Solving (1) and (2), we have $x = 6$, $y = 11$

48. $6\times 18+7\times 24+8k+9\times 20+10\times 13=7.86(18+24+k+20+13)$,
$8k + 586 = 7.86(k + 75)$, $8k – 7.86k = 589.5 – 586$, $\therefore$ $k = 25$

49. Least possible value of k = (18 + 24 – 20 – 13) + 1 = 10

61. Sets I and III are evenly distributed, while “20” is an extreme datum in set II.

64. $\because$ $x+y=2a$, $y+z=2b$, $x+z=2c$,
$\therefore$ $(x+y)+(y+z)+(x+z)=2a+2b+2c$,
$2(x+y+z)=2(a+b+c)$, $x+y+z=a+b+c$,
$\therefore$ mean = $\frac{x+y+z}{3}=\frac{a+b+c}{3}$