The explanations in this unit are in a very concise form, e.g. '∠ sum of Δ' written as 'Δ sum'.
此練習內的解釋較精簡，例如「三角形內角和」會寫成「三角和」。

1-1

1. (a) $OP = \sqrt{3^2 + 4^2} = 5\text{ cm}$; $MN = 5 - 3 = 2\text{ cm}$.
   (b) $OM = \sqrt{10^2 - 8^2} = 6\text{ cm}$; $MN = 10 - 6 = 4\text{ cm}$.
   (c) $PM = \sqrt{8^2 - 6^2} = 5.29\text{ cm}$, $PQ = 10.6\text{ cm}$.
2. (a) $r = \sqrt{5^2 + 6^2} = 7.81\text{ cm}$; $MN = 7.81 - 5 = 2.81\text{ cm}$.
   (b) $AM = \sqrt{8^2 - 3^2} = 7.42\text{ cm}$, $AB = 14.8\text{ cm}$.
3. (a) $4^2 + (r-2)^2 = r^2$, $r = 5$.
   (b) $6^2 + (r-3)^2 = r^2$, $r = 7.5$.
4. (a) $AO_1 = r\text{ cm}$, $O_1M = \frac{1}{2}r\text{ cm}$.
   (b) $AM = \frac{1}{2}AB = 5\sqrt{3}\text{ cm}$, $(5\sqrt{3})^2 + (\frac{1}{2}r)^2 = r^2$, $r = 10$.
5. Join AM, and let its mid-point be M. 連接 AM; 設它的中點為 M。
   $\therefore \angle AMP = 90°$; $PM^2 + AM^2 = AP^2$, $(\frac{\sqrt{108}}{2})^2 + (\frac{1}{2}r)^2 = r^2$, $r = 6$.

1-2

1. (a) $OQ = OA = 2 + 4 = 6\text{ cm}$; $CQ = \sqrt{6^2 - (\sqrt{11})^2} = 5\text{ cm}$.
   (b) $CE = \sqrt{4^2 - (\sqrt{11})^2} = 2.24\text{ cm}$, $PE = PC - CE = 5 - 2.24 = 2.76\text{ cm}$.
2. (a) $BA = BD = 3 + 1 = 4\text{ cm}$, $OA = \sqrt{4^2 + 4^2} = 5.66\text{ cm}$.
   (b) $OC = \sqrt{4^2 + 3^2} = 5\text{ cm}$, $CH = OH - OC = 5.66 - 5 = 0.657\text{ cm}$.
3. (a) (i) $NQ = \frac{4 + 20}{2} = 12\text{ cm}$, $MN = 12 - 4 = 8\text{ cm}$. (ii) $OM = \sqrt{5^2 + 8^2} = 9.43\text{ cm}$.
   (iii) $OQ = \sqrt{5^2 + 12^2} = 13\text{ cm}$, $AM = \sqrt{13^2 - 9.43^2} = 8.94\text{ cm}$,
   $AB = 2AM = 17.9\text{ cm}$.
   (b) (i) $AM = \frac{1}{2}AB = 4\text{ cm}$, $OM = \sqrt{5^2 - 4^2} = 3\text{ cm}$.
   (ii) $MN = \sqrt{3^2 - 1^2} = 2.83\text{ cm}$.
   (iii) $NQ = \sqrt{5^2 - 1^2} = 4.90\text{ cm}$; $PM = PN - MN = 4.90 - 2.83 = 2.07\text{ cm}$.

1-3

1. (a) (i) $OA = OB = \frac{1}{2}AB = 13\text{ cm}$; $OD = OF = 13\text{ cm}$.
   (ii) $CE = ED = \frac{1}{2}CD = 5\text{ cm}$.
   (iii) $OE = \sqrt{13^2 - 5^2} = 12\text{ cm}$; $EF = 13 - 12 = 1\text{ cm}$.

(iv) $AE=\sqrt{13^2+12^2}=17.7$ cm.

(b) (i) $r=\frac{1}{2}AB=11$ cm, $OE=\sqrt{13^2-11^2}=6.93$ cm, $ED=\sqrt{11^2-6.93^2}=8.54$ cm, $CD=2ED=17.1$ cm.

(ii) $EF=11-6.93=4.07$ cm. (iii) $DF=\sqrt{8.54^2+4.07^2}=9.46$ cm.

2. $AF=\frac{1}{2}AB=6$ cm, $CE=\frac{1}{2}CD=7$ cm,
$OE^2+7^2=r^2$ .....(1); $(OE+1)^2+6^2=r^2$ .....(2); $\therefore OE=6$ cm, $r=9.22$.

3. (a) (i) $MB=\frac{1}{2}AB=5$ cm; $ND=\frac{1}{2}CD=4$ cm. (ii) $OM=\sqrt{6^2-5^2}=3.32$ cm.

(iii) $ON=\sqrt{6^2-4^2}=4.47$ cm, $OP=\sqrt{3.32^2+4.47^2}=5.57$ cm.

(b) (i) $MB=\frac{2+4}{2}=3$ cm, $PM=AM-AP=3-2=1$ cm.

(ii) $ND=\frac{1+8}{2}=4.5$ cm, $PN=CN-CP=4.5-1=3.5$ cm.

(iii) In 在 ΔOMB, $r=\sqrt{3.5^2+3^2}=4.61$ cm.

1-4

1. (a) $AM=MB=5$ cm, $CM=\sqrt{26^2-5^2}=25.5$ cm, $OM=(25.5-r)$ cm,
$(25.5-r)^2+5^2=r^2$, $r=13.2$ cm.

(b) $MB=\sqrt{9^2-7^2}=5.66$ cm, $OM=(7-r)$ cm, $(7-r)^2+5.66^2=r^2$, $r=5.79$ cm

(c) $AM=MB=\sqrt{r^2-6^2}$ cm, $(r^2-6^2)+(r+6)^2=18^2$, $r^2+6r-162=0$, $r=10.1$ cm

2. $BE^2+OE^2=17^2$ .....(1); $(BE+12)^2+OE^2=25^2$ .....(2); $\therefore BE=8$ cm, $OE=15$ cm.

1-5

1. $\angle OAB=70^\circ$ (alt. ∠s 內錯角, BA//OD); $x=180^\circ-2\times70^\circ=40^\circ$;
$\angle OCB=y$ (isos. Δ 等腰三角); $x=2y$ (ext. ∠ 外角); $y=20^\circ$.

2. $\angle OQR=66^\circ$ (alt. ∠s 內錯角, PO//QR); $\angle ORQ=66^\circ$ (isos. Δ 等腰三角);
$\angle SOP=66^\circ$ (corr. ∠s 同位角, PO//QR); $\angle PSO=a$ (isos. Δ 等腰三角);
$a=\frac{180^\circ-66^\circ}{2}=57^\circ$.

3. $\angle OMN=24^\circ$ (alt. ∠s 內錯角, OM//NT); $\angle ONM=24^\circ$ (isos. Δ 等腰三角);
$\angle OTN=\angle ONT=48^\circ$ (isos. Δ 等腰三角); $\theta=24^\circ+48^\circ=72^\circ$ (ext. ∠ 外角).

4. $\angle DAB=42^\circ$ (corr. ∠s 同位角, OC // AB).
Join 連接 OB. $\because$ OA = OB, $\therefore \angle AOB=180^\circ-2(42^\circ)=96^\circ$
$\angle BOC=180^\circ-\angle COD-\angle AOB=180^\circ-42^\circ-96^\circ=42^\circ$
$\because$ OB = OC, $\therefore n=\frac{180^\circ-\angle BOC}{2}=\frac{180^\circ-42^\circ}{2}=69^\circ$

5. $\angle OQR=\frac{180^\circ-92^\circ}{2}=44^\circ$ (isos. Δ 等腰三角); $\angle POQ=44^\circ$ (alt. ∠s 內錯角, PO//QR);
$\angle POR=44^\circ+92^\circ=136^\circ$, $\angle OPR=\frac{180^\circ-136^\circ}{2}=22^\circ$ (isos. Δ 等腰三角);

$\theta = 22°$ (alt. $\angle$s 內錯角, PO//QR).

1-6

1. (a) a (b) r (c) q (d) a, b
2. (a) $y = 2 \times 100° = 200°$ (centre & circum.$\angle$s 圓心及圓周角);
   $x = 360° - 200° = 160°$ (同頂角).
   (b) $a = 2 \times 78° = 156°$ (centre & circum.$\angle$s 圓心及圓周角);
   $b = \dfrac{180° - 156°}{2} = 12°$ (isos. Δ 等腰三角).
   (c) $\angle DOE = 2 \times 40° = 80°$ (centre & circum.$\angle$s 圓心及圓周角);
   $\theta = \dfrac{180° - 80°}{2} = 50°$ (isos. Δ 等腰三角).
   (d) $\angle POM = 180° - 38° = 142°$ (int. $\angle$s 內對角, PO//NM);
   reflex 反角 $\angle POM = 360° - 142° = 218°$ ($\angle$s at a pt. 同頂角);
   $x = 218° \div 2 = 109°$ (centre & circum.$\angle$s 圓心及圓周角).

1-7

1. (a) $\angle ACD$ (b) $\angle MSP$, $\angle MRP$ (c) $\angle PQT$, $\angle PRT$
2. (a) $\angle BCD = 90°$ (semi-circle 半圓); $\angle OCB = 35°$ (isos. Δ 等腰三角);
   $\theta = 90° - 35° = 55°$.
   (b) $\angle CFE = 90°$ (semi-circle 半圓); $\angle EFD = 90° - 68° = 22°$;
   $x = 2 \times 22° = 44°$ (centre & circum.$\angle$s 圓心及圓周角).
   (c) $\angle ORS = \dfrac{180° - 114°}{2} = 33°$; $\therefore \angle Q = 33°$;
   $\angle P = 90°$ (semi-circle 半圓); $\alpha = 180° - 90° - 33° = 57°$.
3. $\angle ACB = 90°$ (semi-circle 半圓);
   $\angle ACD = 27° \div 2 = 13.5°$ (centre & circum.$\angle$s 圓心及圓周角);
   $\angle BCD = 90° + 13.5° = 103.5°$.

1-8

1. (a) $\angle CED = 67°$ ($\angle$ in alt. seg. 交錯弓形角); $\theta = 67° + 34° = 101°$ (ext. $\angle$ 外角).
   (b) $\angle PQS = 150° \div 2 = 75°$ (centre & circum.$\angle$s 圓心及圓周角);
   $a = 90° - 75° = 15°$ (ext. $\angle$ 外角); $b = a = 15°$ (equal segment $\angle$s 相等同弓角).
   (c) $\angle MNH = 51°$ (alt. $\angle$s 內錯角, MN//HK);
   $\angle HKP = \angle MNH = 51°$ (segment $\angle$s 同弓角);
   $s = 180° - 51° - 51° = 78°$ (Δ sum 三角和).
   (d) $\angle RSQ = \angle RPQ = 65°$ (segment $\angle$s 同弓角);
   $\angle SRQ = 90°$ (semi-circle 半圓); $\therefore y = 180° - 90° - 65° = 25°$ (Δ sum 三角和).
   (e) $\angle ECG = 180° - 113° - 56° = 11°$ (int. $\angle$s 內對角, CD//EF);
   $\theta = \angle ECG = 11°$ (segment $\angle$s 同弓角).
   (f) Let 設 $a = \angle ODB = \angle OBD$ (isos. Δ 等腰三角);
   $\therefore a + a = 30°$ (ext. $\angle$ 外角); $a = 15°$. $\theta = \angle DCA = \angle DBA = a = 15°$

2. (a) $\angle BAD = 90°$ (semi-circle 半圓); $\angle BDA = \frac{180° - 90°}{2} = 45°$ (isos. Δ 等腰三角);
$\theta = \angle BDA = 45°$ (segment ∠s 同弓角).
(b) $\angle EDF = 26°$ (segment ∠s 同弓角); $\angle CDF = 90°$ (semi-circle 半圓);
$\angle CDE = 90° + 26° = 116°$, $\angle CED = \frac{180° - 116°}{2} = 32°$ (isos. Δ 等腰三角);
$\theta = \angle CED = 32°$ (segment ∠s 同弓角).
(c) $\angle DBC = 78°$ (segment ∠s 同弓角); $\angle ACB = 78°$ (isos. Δ 等腰三角);
$\angle BCD = 90°$ (semi-circle 半圓); $\theta = 90° - 78° = 12°$

1-9

1. (a) $\angle BDC = \angle BAC = 28°$; $\theta = \angle BDC + 63° = 28° + 63° = 91°$.
(b) $\angle ABD = 90°$ (semi-circle 半圓); $\angle CBD = 90° - 39° = 51°$;
$\angle DAC = 51°$ (segment ∠s 同弓角); $x = 51° + 65° = 116°$ (ext. ∠ 外角).
(c) $\angle AOB = 32° - 14° = 18°$ (ext. ∠ 外角); reflex $\angle BOD = 18° + 180° = 198°$;
$y = 198° \div 2 = 99°$ (centre & circum.∠s 圓心及圓周角).
(d) $\angle AOB = 2 \times 25° = 50°$ (centre & circum.∠s 圓心及圓周角);
$\angle OAB = \frac{180° - 50°}{2} = 65°$ (isos. Δ 等腰三角);
$\angle OAC = 25°$ (alt. ∠s 內錯角, OA//CB); $\theta = 65° - 25° = 40°$.
(e) reflex $\angle BOD = 360° - 144° = 216°$ (同頂角);
$x = 216° \div 2 = 108°$ (centre & circum.∠s 圓心及圓周角);
$y = 180° - 108° - 24° = 48°$ (int. ∠s 內對角, AB//DC);
(f) $\angle AOB = 2\theta$ (centre & circum.∠s 圓心及圓周角); $\angle CAD = \theta - 20°$ (ext. ∠ 外角);
$2(47° + \theta - 20°) + 2\theta = 180°$ (Δ sum 三角和); $\theta = 31.5°$

2. (a) $\angle BCA = 180° - n$ (adj. ∠s 鄰角);
reflex $\angle AOB = 2(180° - n) = 360° - 2n$ (centre & circum.∠s 圓心及圓周角);
$m = 360° - (360° - 2n) = 2n$ (同頂角).
(b) $\angle AED = c$ (segment ∠s 同弓角);
$\angle EDC = a + \angle AED = a + c$ (ext. ∠ 外角);
$b = \angle EDC + c = (a + c + c) = a + 2c$ (ext. ∠ 外角); $\therefore b = a + 2c$
(c) $\angle CAD = y$ (segment ∠s 同弓角); $\angle ADC = 90°$ (semi-circle 半圓);
$x + y + 90° = 180°$ (Δ sum 三角和); $x + y = 90°$.
(d) $\angle AOB = 360° - a$ (同頂角);
$\angle ACB = \frac{360° - a}{2}$ (centre & circum.∠s 圓心及圓周角);
$a + b + c + \frac{360° - a}{2} = 360°$ (∠sum 內角和); $a + 2b + 2c = 360°$.
(e) $\angle BAC = q - p$ (ext. ∠ 外角);
$\angle BOC = 2(q - p) = 2q - 2p$ (centre & circum.∠s 圓心及圓周角);
$2q - 2p + r = q$ (ext. ∠ 外角); $q - 2p + r = 0$.
(f) $\angle ADC = x$ (segment ∠s 同弓角); $\angle ABD = \angle ADC = x$ (isos. Δ 等腰三角);
$\angle DCE = 2x$ (ext. ∠ 外角); $y = x + 2x = 3x$ (ext. ∠ 外角).

1-10

1. (a) T (b) F (c) F
2. (a) F (b) T (c) F
3. (a) T (b) F (c) T
4. (a) F (b) T (c) T
5. (a) $a = 49°$, $b = 180° - 49° = 131°$ (opp. ∠s, cyclic quad. 圓對角).
   (b) $\angle QPS = 180° - 65° = 115°$ (int. ∠s 內對角, PQ//SR);
   $x = 180° - 115° = 65°$ (opp. ∠s, cyclic quad. 圓對角).
   (c) $2x = 86°$ (ext. ∠, cyclic quad. 圓外角); $x = 43°$;
   $y = x = 43°$ (segment ∠s 同弓角).
   (d) $\angle CEB = 46°$ (segment ∠s 同弓角);
   $x = 180° - 35° - 46° - 87° = 12°$ (Δ sum 三角和);
   $y = 87° - 12° = 75°$ (ext. ∠, cyclic quad. 圓外角).
   (e) $\angle MPN = 90°$ (semi-circle 半圓);
   $\angle PMN = 180° - 117° = 63°$ (opp. ∠s, cyclic quad. 圓對角);
   $\theta = 180° - 90° - 63° = 27°$ (Δ sum 三角和).
   (f) Join 連接 QT. $\angle SQT = 90°$ (semi-circle 半圓); $\therefore \angle RQT = 44° + 90° = 134°$;
   $\angle QTP + 29° = \angle RQT$ (ext. ∠ 外角); $\therefore \angle QTP = 134° - 29° = 105°$
   $a = \angle QTP = 105°$ (ext. ∠, cyclic quad. 圓外角).
6. (a) $\angle PSR = 180° - 97° = 83°$ (opp. ∠s, cyclic quad. 圓對角);
   $\angle PST = 90°$ (semi-circle 半圓); $\theta = 83° + 90° = 173°$.
   (b) $\angle SPQ = 180° - 102° = 78°$ (opp. ∠s, cyclic quad. 圓對角);
   $\theta = \angle SPQ = 78°$ (ext. ∠, cyclic quad. 圓外角).
   (c) $\angle BED = 180° - 105° = 75°$ (opp. ∠s, cyclic quad. 圓對角);
   $\angle BDE = 180° - 33° - 75° = 72°$ (Δ sum 三角和);
   $x = 180° - 72° = 108°$ (opp. ∠s, cyclic quad. 圓對角);
   (d) $\angle HOL = 180° - 34° = 146°$ (opp. ∠s, cyclic quad. 圓對角);
   $\theta = 146° \div 2 = 73°$ (centre & circum.∠s 圓心及圓周角).
7. $\angle ABF = p + q$ (ext. ∠ 外角); $\angle BED = r + p + q$ (ext. ∠ 外角);
   $\angle BED + p = 180°$ (opp. ∠s, cyclic quad. 圓對角); $2p + q + r = 180°$
8. $\angle B + \angle AFC = 180°$ (opp. ∠s, cyclic quad. 圓對角);
   $\angle D + \angle EFC = 180°$ (opp. ∠s, cyclic quad. 圓對角);
   $\angle B + \angle D + \angle F = \angle B + \angle AFC + \angle D + \angle EFC = 180° + 180° = 360°$
9. $\angle CDP = y$ (ext. ∠, cyclic quad. 圓外角);
   reflex $\angle COP = 2y$ (centre & circum.∠s 圓心及圓周角); $x = 360° - 2y$ (同頂角).
10. $\angle ABQ = 96°$ (ext. ∠, cyclic quad. 圓外角);
   $b = 180° - 96° = 84°$ (opp. ∠s, cyclic quad. 圓對角);
   $\angle AOQ = 2 \times 84° = 168°$ (centre & circum.∠s 圓心及圓周角);
   $a = 360° - 40° - 168° - 96° = 56°$ (∠sum 內角和).

1-11

1. $\angle DAE = \angle CAB$ (common ∠ 共角); $\angle AED = \angle ABC$ (ext. ∠, cyclic quad. 圓外角);
   $\therefore$ $\Delta AED \sim \Delta ABC$ (A.A.A.); $\frac{AE}{AB} = \frac{AD}{AC} = \frac{ED}{BC}$ (corr. sides 對應邊, ~Δs);
   $\frac{a}{5+b} = \frac{5}{a+4} = \frac{8}{16}$, $\therefore a = 2 \times 5 - 4 = 6$, $b = 2 \times 6 - 5 = 7$.

2. $\angle RPQ = \angle HPK$ (common ∠ 共角); $\angle PQR = \angle PKH$ (ext. ∠, cyclic quad. 圓外角);
$\therefore$ $\Delta PQR \sim \Delta PKH$ (A.A.A.); $\dfrac{PQ}{PK} = \dfrac{PR}{PH} = \dfrac{QR}{KH}$ (corr. sides 對應邊, ~Δs);
$\dfrac{12+15}{9} = \dfrac{y+9}{12} = \dfrac{x}{10}$, $\therefore x = 3 \times 10 = 30$, $y = 3 \times 12 - 9 = 27$.
3. $\angle CAD = \angle EAB$ (common ∠ 共角); $\angle ACD = \angle AEB$ (ext. ∠, cyclic quad. 圓外角);
$\therefore$ $\Delta ACD \sim \Delta AEB$ (A.A.A.); $\dfrac{AC}{AE} = \dfrac{AD}{AB}$ (corr. sides 對應邊, ~Δs); $\dfrac{12+x}{14} = \dfrac{14+16}{12}$,
$\therefore x = 2.5 \times 14 - 12 = 23$.
4. $\angle SPR = \angle QPT$ (common ∠ 共角); $\angle PRS = \angle PTQ$ (ext. ∠, cyclic quad. 圓外角);
$\therefore$ $\Delta PRS \sim \Delta PTQ$ (A.A.A.); $\dfrac{PR}{PT} = \dfrac{PS}{PQ}$ (corr. sides 對應邊, ~Δs); $\dfrac{y+4}{12} = \dfrac{12+9}{y}$,
$\therefore y^2 + 4y - 252 = 0$, $y = 14$ or 或 $y = -18$ (rejected 捨去).

1-12

1. (a) $\angle SRT = 180° - 34° - 56° = 90°$ (Δ sum 三角和); $\angle P + \angle SRT = 90° + 90° = 180°$
$\therefore$ P, Q, R, S are concyclic. P、Q、R、S 共圓。 (opp. ∠s supp. 對角互補)
(b) $\angle P = 180° - 27° - 50° - 67° = 36°$ (Δ sum 三角和); $\angle P = \angle Q = 36°$,
$\therefore$ P, Q, R, S are concyclic. P、Q、R、S 共圓。 (equal segment ∠s 相等同弓角).
(c) $\angle PSQ = 138° - 83° = 55°$ (ext. ∠ 外角); $\angle PSQ = \angle PRQ = 55°$,
$\therefore$ P, Q, R, S are concyclic. P、Q、R、S 共圓。 (equal segment ∠s 相等同弓角).
(d) $\angle P = 63° - 21° = 42°$ (ext. ∠ 外角);
$\angle PQR = 180° - 75° - 42° = 63°$ (Δ sum 三角和); $\angle LSR = \angle PQR = 63°$,
$\therefore$ P, Q, R, S are concyclic. P、Q、R、S 共圓。 (ext.∠外角 = int. opp.∠內對角).
2. (a) $\angle ABD = 180° - 35° - 46° - 39° = 60°$ (Δ sum 三角和); $\angle AEH = \angle ABD = 60°$,
$\therefore$ B, A, G, E are concyclic. B、A、G、E 共圓。 (equal segment ∠s 相等同弓角).
(b) $\angle EFG = 46° + 39° = 85°$ (ext. ∠ 外角); $\angle FGH = 85° - 60° = 145°$ (ext. ∠ 外角);
$\angle FAH + \angle FGH = 46° + 145° = 191° \neq 180°$,
$\therefore$ A, F, G, H are not concyclic. A、F、G、H 不是共圓。
(c) $\angle EFG + \angle C = 85° + 95° = 180°$,
$\therefore$ C, D, F, E are concyclic. C、D、E、F 不是共圓。(opp. ∠s supp. 對角互補).
3. $\angle PQA + \angle D = 180°$ (opp. ∠s, cyclic quad. 圓對角);
$\angle C + \angle D = 180°$ (int. ∠s 內對角, BC//AD); $\angle PQA = \angle C$,
$\therefore$ B, C, P, Q are concyclic. P、Q、R、S 是共圓。 (ext.∠外角 = int. opp.∠內對角).
4. (a) $4x° + 6x° + 5x° + 3x° = 360°$ (∠sum 內角和); $x = 20$
(b) $\angle Q + \angle S = 4 \times 20° + 5 \times 20° = 180°$,
$\therefore$ PQRS is a cyclic quad. P、Q、R、S 是圓內接四邊形。 (opp. ∠s supp. 對角互補).
5. (a) $\angle BMC = \angle CNB = 90°$, $\therefore$ BCMN is a cyclic quad. B、C、M、N 是圓內接四邊形。
(equal segment ∠s 相等同弓角).
(b) $\angle BMN = 180° - 90° - 68° = 22°$ (adj. ∠s 鄰角);
$\theta = \angle BMN = 22°$ (segment ∠s 同弓角).
6. $\angle QPR = 180° - 106° - 22° = 52°$ (∠sum 內角和); $\angle QSR = \angle QPR = 52°$,
$\therefore$ PQRS is a cyclic quad. P、Q、R、S 是圓內接四邊形。 (equal segment ∠s 相等同弓角).
$\theta = \angle PRQ = 22°$ (segment ∠s 同弓角).

1-13

1. (a) $\angle ACD$, $\angle CAD$ (b) No (c) $\angle SRT$, $\angle SPT$

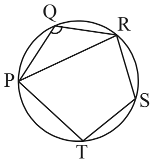
2. (a) $\angle BAC = \angle CAD = x$ (eq. arcs 等弧);
$\therefore x + x + 74° + 50° = 180°$ (Δ sum 三角和); $x = 28°$.
(b) $\angle BPC = 46°$ (eq. chords 等弦);
$x = 180° - 46° - 46° - 38° = 50°$ (opp. $\angle$s, cyclic quad. 圓對角).
(c) $\angle PRQ = \theta$ (eq. arcs 等弧);
$\angle PQR = 180° - 82° = 98°$ (opp. $\angle$s, cyclic quad. 圓對角);
$\theta = \frac{180° - 98°}{2} = 41°$.
(d) $\angle CAB = 35°$ (eq. arcs 等弧); $\angle ACB = 90°$ (semi-circle 半圓);
$\theta = 180° - 35° - 90° = 55°$ (Δ sum 三角和).
(e) $\angle QOR = 34°$ (eq. arcs 等弧); $\angle POR = 34° + 34° = 68°$;
$\theta = 68° \div 2 = 34°$ (centre & circum.$\angle$s 圓心及圓周角).
(f) $\angle EAC = 180° - 126° = 54°$ (opp. $\angle$s, cyclic quad. 圓對角);
$\angle BAC : 54° = \overset{\frown}{BC} : \overset{\frown}{CE} = 1 : 2$ (arc prop.弧比例);
$\angle BAC = 27°$, $y = 27° + 54° = 81°$.

3. (a) $\angle OAB = 40°$ (isos. Δ 等腰三角); $x = 180° - 2 \times 40° = 100°$ (Δ sum 三角和);
$y = 40°$ (alt. $\angle$s 內錯角, AB//DC); $\overset{\frown}{AB} : \overset{\frown}{BC} = 100° : 40° = 5 : 2$ (arc prop.弧比例);
(b) $a = 100° - 25° = 75°$ (ext. $\angle$ 外角); $\overset{\frown}{BC} : \overset{\frown}{AD} = 25° : 75° = 1 : 3$ (arc prop.弧比例).
(c) $\angle ADB = 90°$ (semi-circle 半圓); $\angle ADC = 90° + 50° = 140°$;
$x = \frac{180° - 140°}{2} = 20°$ (isos. Δ 等腰三角);
$y = 180° - 90° - 20° = 70°$ (Δ sum 三角和);
$\overset{\frown}{AD} : \overset{\frown}{DB} = 70° : 20° = 7 : 2$ (arc prop.弧比例).
(d) $b = 180° - 70° = 110°$ (opp. $\angle$s, cyclic quad. 圓對角);
$a = 180° - 110° - 35° = 35°$ (Δ sum 三角和);
$\overset{\frown}{PS} : \overset{\frown}{SR} = 35° : 35° = 1 : 1$ (arc prop.弧比例);
$\overset{\frown}{PQR} : \overset{\frown}{PSR} = 110° : 70° = 11 : 7$ (arc prop.弧比例);
(e) $\alpha = 180° - 102° - 26° = 52°$ (Δ sum 三角和);
$\angle ACD = 90°$ (semi-circle 半圓);
$\theta = 180° - 52° - 90° - 26° = 12°$ (opp. $\angle$s, cyclic quad. 圓對角);
$\overset{\frown}{AB} : \overset{\frown}{BC} : \overset{\frown}{CD} = 52° : 26° : 12° = 26 : 13 : 6$ (arc prop.弧比例).
(f) $\angle PTQ = 90°$ (semi-circle 半圓); $\angle PQT = 15° + 25° = 40°$ (ext. $\angle$ 外角);
$\theta = 180° - 90° - 15° - 40° = 35°$ (opp. $\angle$s, cyclic quad. 圓對角);
$\overset{\frown}{PT} : \overset{\frown}{TS} = 40° : 35° = 8 : 7$ (arc prop.弧比例).

1-14

1. $\angle ADC = 180° - 81° = 99°$ (opp. ∠s, cyclic quad. 圓對角);

$\angle ACB : 99° = \overset{\frown}{AB} : \overset{\frown}{AC} = 2:3$ (arc prop.弧比例); $\angle ACB = 66°$.

2. 

Join 連結 PR.

$\angle Q : \angle QPR : \angle QRP = \overset{\frown}{RSP} : \overset{\frown}{QR} : \overset{\frown}{PQ} = 10:2:3$ (arc prop.弧比例);

Let $\angle Q = 10k$, $\angle QPR = 2k$, $\angle QRP = 3k$

$10k + 2k + 3k = 180°$ (Δ sum 三角和); $k = 12°$. ∴

$\angle Q = 10 \times 12° = 120°$.

3. (a) $\angle AOB : 90° = AB : AD = 1:3$ (arc prop.弧比例); $\angle AOB = 30°$

$\angle ADB = 30° \div 2 = 15°$ (centre & circum.∠s 圓心及圓周角).

(b) $\angle BOD = 60°$, $\angle BDO = \dfrac{180° - 60°}{2} = 60°$. Similarly, $\angle CAO = 60°$.

$x = 360° - 90° - 60° - 60° = 150°$ (Δ sum 三角和).

1-15

1. (a) $\angle BAO = \dfrac{180° - 106°}{2} = 37°$ (isos. Δ 等腰三角);

$\theta = 90° - 37° = 53°$ (tangent 切線 ⊥ r).

(b) $AC = BC$ (tangent 切線); $x = 180° - 2 \times 63° = 54°$ (isos. Δ 等腰三角);

$\angle OAB = 90° - 63° = 27°$ (tangent 切線 ⊥ r);

$y = 180° - 2 \times 27° = 126°$ (isos. Δ 等腰三角).

(c) $\angle DTF = 90° - 58° = 32°$ (tangent 切線 ⊥ r); $\theta = 32°$ (alt. ∠s 內錯角, TD//EF).

(d) $\angle ONL = \dfrac{180° - 82°}{2} = 49°$ (isos. Δ 等腰三角);

$\angle TOM = 49°$ (alt. ∠s 內錯角, TO//NL); $\angle OTM = 90°$ (tangent 切線 ⊥ r);

$x = 180° - 90° - 49° = 41°$ (Δ sum 三角和).

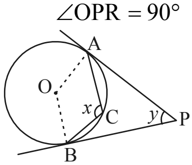
(e) $\angle TPR = 90°$ (tangent 切線 ⊥ r); $y = 180° - 90° - 74° = 16°$ (Δ sum 三角和);

$TP = TS$ (tangent 切線); $\angle TSP = \dfrac{180° - 74°}{2} = 53°$ (isos. Δ 等腰三角);

$\angle PSQ = 90°$ (semi-circle 半圓); $x = 180° - 90° - 53° = 37°$ (adj. ∠s 鄰角).

(f) $\angle PQS = 77° \div 2 = 38.5°$ (centre & circum.∠s 圓心及圓周角);

$\angle OPR = 90°$ (tangent 切線 ⊥ r); $\theta = 180° - 90° - 38.5° = 51.5°$ (Δ sum 三角和).

2. 

reflex $\angle AOB = 2x$ (centre & circum.∠s 圓心及圓周角);

$\angle AOB = 360° - 2x$ (同頂角);

$\angle OAP = \angle OBP = 90°$ (tangent 切線 ⊥ r);

$360° - 2x + 90° + 90° + y = 360°$ (∠sum 內角和); $x = 90° + \dfrac{y}{2}$.

3. $TP = TQ$ (tangent 切線); $TP = TR$ (tangent 切線); ∴ $TQ = TR$.

$\angle PTQ = 180° - 2 \times 75° = 30°$ (isos. Δ 等腰三角);

$\angle QTR = 48° - 30° = 18°$; $\theta = \dfrac{180° - 18°}{2} = 81°$ (isos. Δ 等腰三角).

1-16

1. $AD = AF = 12\text{ cm}$, $BE = BD = 8\text{ cm}$, $FC = EC = 6\text{ cm}$ (tangent 切線);
   perimeter 周界 $= 12\times2+8\times2+6\times2 = 52\text{ cm}$.
2. $QB = QR = 5\text{ cm}$, $AP = RP = 9\text{ cm}$ (tangent 切線);
   $TA = TB$ (tangent 切線); $TP + 9 = 14 + 5$, $TP = 10\text{ cm}$.
3. $\angle OTQ = 90°$ (tangent 切線 ⊥ r); $r^2 + 24^2 = (r+16)^2$, $r = 10\text{ cm}$.
4. $\angle ACD = \angle BDC = 90°$ (tangent 切線 ⊥ r);
   Draw a ⊥ line from A to BD. 由 A 畫垂直線至 BD.
   $\therefore AB^2 = 24^2 + (17-7)^2$; $AB = 26\text{ cm}$.

1-17

1. (a) $\angle THK = 38°$ (∠ in alt. seg. 交錯弓形角);
   $b = 180° - 65° - 38° = 77°$ (Δ sum 三角和).
   (b) $AB = AC$ (tangent 切線); $x = \dfrac{180° - 72°}{2} = 54°$ (isos. Δ 等腰三角);
   $y = x = 54°$ (∠ in alt. seg. 交錯弓形角).
   (c) $\angle CFE = 90°$; $\therefore \angle EFA = 90° - 37° = 53°$;
   $m = \angle EFA = 53°$ (∠ in alt. seg. 交錯弓形角).
   (d) $a = 68°$ (∠ in alt. seg. 交錯弓形角); $b = 51°$ (∠ in alt. seg. 交錯弓形角);
   $c = 97° - 51° = 46°$ (ext. ∠ 外角).
   (e) Join 連結 QP.
   $\angle TQP = 42°$ (∠ in alt. seg. 交錯弓形角); $\therefore \angle OQP = 65° - 42° = 23°$;
   $\because OQ = OP$; $\therefore k = 180° - 23°\times2 = 134°$ (isos. Δ 等腰三角).
   (f) $\angle KFG = 48°$ (∠ in alt. seg. 交錯弓形角);
   $\therefore b = 180° - 24° - 48° = 108°$ (Δ sum 三角和);
   $a = 180° - b = 72°$ (opp. ∠s, cyclic quad. 圓對角).
   (g) $\angle LAN = 180° - 82° = 98°$ (opp. ∠s, cyclic quad. 圓對角);
   $\angle NLA = 70°$ (∠ in alt. seg. 交錯弓形角);
   $\theta = 180° - 70° - 98° = 12°$ (Δ sum 三角和).
   (h) $\angle ABE = 69°$ (∠ in alt. seg. 交錯弓形角); $\angle BEF = 69°$ (alt. ∠s 內錯角, AB//DF);
   $\angle BEC = 69° - 37° = 32°$; $\theta = 32°$ (∠ in alt. seg. 交錯弓形角).
   (i) $\angle QTR = 90°$ (semi-circle 半圓); $\angle QTP = \theta$ (∠ in alt. seg. 交錯弓形角);
   $46° + \theta + 90° + \theta = 180°$ (Δ sum 三角和); $\theta = 22°$
2. $\angle PRQ = 59°$ (corr. ∠s 同位角, BA//RQ);
   $\angle APQ = \angle PRQ = 59°$ (∠ in alt. seg. 交錯弓形角);
   $AP = AQ$ (tangent 切線); $a = \angle APQ = 59°$ (isos. Δ 等腰三角);
   $b = 180° - 2\times59° - 57° = 5°$ (Δ sum 三角和).
3. $\angle CDA = \angle CAD = 55°$ (∠ in alt. seg. 交錯弓形角);
   $y = 55° + 55° = 110°$ (ext. ∠ 外角); $\angle BAC = 90°$ (tangent 切線 ⊥ r);
   $x = 110° - 90° = 20°$ (ext. ∠ 外角).
4. $\angle BAT : b = \overset{\frown}{BT} : \overset{\frown}{CT} = 3:1$ (arc prop. 弧比例); $\angle BAT = 3b$;
   $\angle ATP = \angle BAT = 3b$ (∠ in alt. seg. 交錯弓形角);
   $a = \angle ATP = 3b$ (alt. ∠s 內錯角, AB//TP).

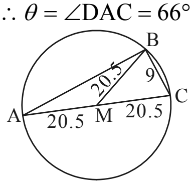
1-18

1. (a) $\angle BCD = 180° - 66° - 45° = 69° \neq 121°$, $\therefore$ AB is not a tangent. AB 不是切線。
   (b) $\angle P = 180° - 88° - 37° = 55°$ (Δ sum 三角和);
   $\angle ABQ = 88° - 33° = 55°$ (ext.∠外角); $\angle P = \angle ABQ = 55°$,
   $\therefore$ AB is a tangent. AB 是切線。(converse of ∠ in alt. seg. 交錯弓形角逆定理).
   (c) $\angle ACD = \angle ADC$ (isos. Δ 等腰三角) $= \angle DAB$ (alt. ∠s 內錯角, CD//AB);
   $\therefore$ AB is a tangent. AB 是切線。(converse of ∠ in alt. seg. 交錯弓形角逆定理).
2. $\angle PQS = \angle SRQ$ (∠ in alt. seg. 交錯弓形角) $= \angle SNM$ (ext. ∠, cyclic quad. 圓外角);
   $\therefore$ QP//MN (alt. ∠s equal 內錯角相等).
3. $\angle ODB = 55°$ (isos. Δ 等腰三角); $\angle ODE = 145° - 55° = 90°$;
   $\therefore$ ED is a tangent. ED 是切線。(converse of tangent ⊥ r 切線半徑逆定理).
   OR 或：
   $\angle ADB = 90°$ (segment ∠s 同弓角); $\angle ADE = 145° - 90° = 55° = \angle ABD$
   $\therefore$ ED is a tangent. ED 是切線。(converse of ∠ in alt. seg. 交錯弓形角逆定理).
4. (a) $\angle LMN = 180° - 32° - 24° = 124°$ (Δ sum 三角和); $\angle TLN = \angle LMN = 124°$,
   $\therefore$ TL is a tangent. TL 是切線。(converse of ∠ in alt. seg. 交錯弓形角逆定理).
   (b) $\angle OLT = 90°$ (tangent 切線 ⊥ r); $\angle OLN = 124° - 90° = 34°$;
   $\theta = \angle OLN = 34°$ (isos. Δ 等腰三角).

1-19

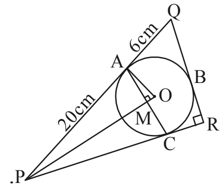
1. $\Delta ABD \sim \Delta ACB$; $\frac{AB}{AC} = \frac{AD}{AB}$, $\frac{x}{4+12} = \frac{4}{x}$, $x = 8$; $\frac{CB}{BD} = \frac{AC}{AB}$, $\frac{y}{10} = \frac{16}{8}$, $y = 20$.
2. $\Delta RQS \sim \Delta RPQ$; $\frac{RS}{RQ} = \frac{QS}{PQ}$, $\frac{n}{12} = \frac{14}{21}$, $n = 8$; $\frac{RP}{RQ} = \frac{PQ}{QS}$, $\frac{m+8}{12} = \frac{21}{14}$, $m = 10$.
3. $\Delta RSQ \sim \Delta RPS$; $\frac{RS}{RP} = \frac{RQ}{RS}$, $\frac{x}{10+8} = \frac{10}{x}$, $x = 13.4$.
4. $\Delta PAC \sim \Delta PBA$; $\frac{PA}{PB} = \frac{PC}{PA}$, $\frac{a}{27+21} = \frac{27}{a}$, $a = 36$.

1-20

1. A circle through A, B, C can be drawn with centre M and radius 20.5.
   可畫圓穿過 A、B、C，圓心為 M，半徑為 20.5。$\angle ABC = 90°$ (semi-circle 半圓);
   $AB = \sqrt{41^2 - 9^2} = 40$; area of ΔABC面積 $= \frac{9 \times 40}{2} = 180$.
2. $\angle ADC = \angle AEC = 90°$, $\therefore$ ACED is a cyclic quad. (equal segment ∠s 相等同弓角);
   $\therefore \theta = \angle DAC = 66°$ (ext. ∠, cyclic quad. 圓外角).
3. 

   $\angle A = (5-2) \times 180° \div 5 = 108°$ (∠sum 內角和);
   $AB = AC$ (tangent 切線);
   $\angle ABC = \angle ACB = \frac{180° - 108°}{2} = 36°$ (isos. Δ 等腰三角);
   $x = 36°$ (∠ in alt. seg. 交錯弓形角).
4. $\angle CKL = \angle CAL = b$ (segment ∠s 同弓角); $\angle CKH = a$ (∠ in alt. seg. 交錯弓形角);
   $\angle CKL + \angle CKH = 180°$ (adj. ∠s 鄰角); $\therefore a + b = 180°$

5. (a) $AB = 52 + 52 = 104$; $\angle ACB = 90°$ (semi-circle 半圓); $x = \sqrt{104^2 - 40^2} = 96$.
   (b) $\angle ACB = 90°$ (semi-circle 半圓); $AB = \sqrt{9^2 + 12^2} = 15$;
   $y = OA = OB = 15 \div 2 = 7.5$.
6. (a) $PC = 20$ cm, $QB = 6$ cm, $RC = RB = x$ cm (tangent 切線);
   $(x+20)^2 + (x+6)^2 = 26^2$; $x^2 + 26x - 120 = 0$, $x = 4$ or $x = -30$ (rejected 捨去);
   $\angle OCR = \angle OBR = 90°$ (tangent 切線 ⊥ r);
   ∴ OBRC is a square. ∴ radius $= OC = x = 4$.
   (b) $\tan\angle AOP = \frac{20}{4}$, $\angle AOP = 78.69°$,
   $OM = 4\cos 78.69° = 0.784$ cm



7. $\angle OAB = 90°$ (tangent 切線 ⊥ r);
   $\angle OMC = 90°$ (mid-pt. of chord 弦中點);
   $\angle OAB + \angle OMC = 90° + 90° = 180°$,
   ∴ OABM is a cyclic quad. OM 是圓內接四邊形(opp. ∠s supp. 對角互補);
   ∴ $a = b$ (segment ∠s 同弓角).

1-21

1. $\Delta APB \sim \Delta DPC$ (A.A.A.); $\therefore \frac{AB}{12} = \frac{12}{6}$, $AB = 24$.
2. $AB = 15$ cm, $\angle BDA = 90°$ (semi-circle 半圓), $\therefore BD = \sqrt{15^2 + 9^2} = 12$ cm,
   area 面積 $= 9 \times 12 = 108$ cm^2.
3. $\angle BOC = 2 \times 30° = 60°$ (centre & circum. ∠ s 圓心及圓周角);
   $\angle OBC = 90°$ (tangent 切線⊥r); $\tan 60° = \frac{12}{r}$, $r = \frac{12}{\sqrt{3}} = 4\sqrt{3}$.
4. $\angle APQ = \angle ARS$, $\angle AQP = \angle ASR$ (ext. ∠, cyclic quad.圓外角); $\angle A$ (common 共角);
   $\therefore \Delta APQ \sim \Delta ARS$ (A.A.A.); $\therefore \frac{3}{4+PS} = \frac{4}{3+9}$, $PS = 5$.
5. $\Delta PAB \sim \Delta CAP$ (A.A.A.) [$\because \angle APB = \angle ACP$ (∠ in alt. seg.交錯弓形角)];
   $\frac{25}{x} = \frac{x}{25+24}$, $x = 35$.
6. $\angle ACB = 90°$ (semi-circle 半圓); $\angle CAB = 28°$ (Δ sum 三角和);
   $\overset{\frown}{AC} : \overset{\frown}{BC} = \angle CBA : \angle CAB = 31:14$ (arc prop.弧比例), $\therefore \overset{\frown}{AC} = 31$ cm.
7. $\angle CBD = 35°$ (isos. Δ 等腰三角);
   $\angle ADB = 180° - 40° - 35° \times 2 = 70°$ (opp. ∠ s, cyclic quad.圓對角);
   $\overset{\frown}{AB} : \overset{\frown}{BC} = 70° : 35° = 2:1$ (arc prop.弧比例).

1-22

1. $\because OC = OD$, $\therefore \angle OCD = 43°$ (isos. Δ 等腰三角);
   $\angle COD = 180° - 43° \times 2 = 94°$ (Δ sum 三角和);
   $a = \frac{1}{2} \times \angle COD = 47°$ (centre & circum. ∠ s 圓心及圓周角).

2. $\because OC = OB$, $\therefore \angle OCB = 38°$ (isos. Δ 等腰三角); $\angle BCA = 15°$,
$\angle BOA = 2\times 15° = 30°$ (centre & circum. ∠ s 圓心及圓周角).
3. reflex 反角 $\angle AOC = 2(x+45°)$ (centre & circum. ∠ s 圓心及圓周角);
$2(x+45°)+3x = 360°$ (∠ s at a pt.同頂角), $x = 54°$; $\angle AOC = 3\times 54° = 162°$.
4. $\angle BDA = x$ (eq. chord, eq. ∠ 等弦等角); $\angle ACD = 90°$ (semi-circle 半圓);
$x+(x+18°)+90° = 180°$ (Δ sum 三角和), $x = 36°$.
5. (a) $\angle BCA = \frac{1}{2}\angle BOA = 27°$ (centre & circum. ∠ s 圓心及圓周角).
(b) $\angle CAO = \angle BCA = 27°$ (alt. ∠ s 內錯角, $AD // BC$), $y = 54°+27° = 81°$ (ext. ∠ 外角).
6. Join 連 CD, let 設 $\angle CAD = \angle BDC = \angle ADB = k$, $\angle ACD = 3k$ (arc prop.弧比例);
$k+k+k+3k = 180°$ (Δ sum 三角和), $k = 30°$; $\angle CAD = 30°$.
7. $76°+a+47° = 180°$ (opp. ∠ s, cyclic quad.圓對角), $a = 57°$;
$\because OD = OC$, $\angle ODC = 57°$ (isos. Δ 等腰三角); $b = 180°-57°\times 2 = 66°$ (Δ sum 三角和).
8. $\angle ABD = 40°$ (segment ∠ s 同弓角); $\angle ACB = 57°$ (Δ sum 三角和);
$\angle BAE = 57°$ (∠ in alt. seg.交錯弓形角).
9. (a) $\angle BOC = \theta$ (isos. Δ 等腰三角);
$\angle DAC = \frac{1}{2}\angle BOC = \frac{1}{2}\theta$ (centre & circum. ∠ s 圓心及圓周角).
(b) $\frac{1}{2}\theta+\theta = 28°$ (ext. ∠ 外角), $\theta = \frac{56°}{3}$.
10. (a) $\angle OAP = \angle OBP = 90°$ (tangent 切線 ⊥ r).
(b) reflex 反角 $\angle AOB = 2\times 107° = 214°$ (centre & circum. ∠ s 圓心及圓周角);
$x = 146°$ (∠ s at a pt.同頂角).
(c) $146°+90°+90°+y = 360°$, $y = 34°$.
11. (a) $\angle CDB = 5x$; $\angle ADB = 8x$ (arc prop.弧比例).
(b) $\angle ABD = 90°$ (semi-circle 半圓);
$90°+2x+5x+8x = 180°$ (opp. ∠ s, cyclic quad.圓對角), $x = 6°$;
$\angle BAD = 180°-90°-48° = 42°$ (Δ sum 三角和).
12. (a) $\angle POR = 2\angle PQR = 96°$ (centre & circum. ∠ s 圓心及圓周角).
(b) circumference 圓周 $= 5\times\frac{360}{96} = 18.75$ cm.
13. (a) $AP = (10-r)$ cm; $QC = (24-r)$ cm.
(b) $AC = \sqrt{10^2+24^2} = 26$ cm; $AR = AP$, $RC = QC$ (tangent 切線);
$(10-r)+(24-r) = 26$, $r = 4$ (cm)

2-1

1. (a) $\cos x = \frac{\sqrt{5}}{3}$, $\tan x = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ (b) $\sin x = \frac{\sqrt{15}}{8}$, $\tan x = \frac{\sqrt{15}}{7}$
(c) $\sin x = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$, $\cos x = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ (d) $\sin x = \frac{3\sqrt{5}}{7}$, $\tan x = \frac{3\sqrt{5}}{2}$
2. (a) I (b) II (c) I (d) II (e) II (f) III
(g) III (h) III (i) IV (j) I (k) IV (l) I
3. (a) IV (b) I (c) II (d) III

2-2

1.

θ		$\sin\theta$	$\cos\theta$	$\tan\theta$
(a)	280°	–	+	–
(b)	202°	–	–	+
(c)	97°	+	–	–
(d)	82°	+	+	+
(e)	257°	–	–	+
(f)	52°	+	+	+
(g)	197°	–	–	+
(h)	342°	–	+	–
(i)	297°	–	+	–
(j)	135°	+	–	–
(k)	248°	–	–	+
(l)	347°	–	+	–

2. (a) + (b) – (c) – (d) – (e) – (f) +
(g) – (h) + (i) + (j) – (k) – (l) –
(m) – (n) + (o) +

3. (a) I (b) IV (c) III (d) I (e) IV (f) I
(g) II (h) I, III (i) I, II (j) II, III (k) II, IV (l) I, IV

2-3

1. ✗ $\sin 235° = \sin(180° + 55°) = -\sin 55°$
2. ✓ $\sin 190° = \sin(180° + 10°) = -\sin 10°$
3. ✓ $\sin 315° = \sin(360° - 45°) = -\sin 45°$; $\sin 225° = \sin(180° + 45°) = -\sin 45°$
4. ✗ $\sin 250° = \sin(180° + 70°) = -\sin 70°$
5. ✓ $-\cos 205° = -\cos(180° + 25°) = -(-\cos 25°) = \cos 25°$;
   $\cos 335° = \cos(360° - 25°) = \cos 25°$
6. ✗ $\cos 240° = \cos(180° + 60°) = -\cos 60°$; $\cos 300° = \cos(360° - 60°) = \cos 60°$
7. ✗ $-\cos 280° = -\cos(360° - 80°) = -\cos 80°$
8. ✓ $-\cos 140° = -\cos(180° - 40°) = -(-\cos 40°) = \cos 40°$
9. ✓ $\tan 154° = \tan(180° - 26°) = -\tan 26°$; $\tan 334° = \tan(360° - 26°) = -\tan 26°$
10. ✓ $-\tan 315° = -\tan(360° - 45°) = -(-\tan 45°) = \tan 45°$
11. ✗ $\tan 190° = \tan(180° + 10°) = \tan 10°$
12. ✓ $\tan 155° = \tan(180° - 25°) = -\tan 25°$; $-\tan 205° = -\tan(180° + 25°) = -\tan 25°$
13. ✗ $\cos 150° = \cos(180° - 30°) = -\cos 30°$; $\cos 240° = \cos(180° + 60°) = -\cos 60°$
14. ✗ $\sin 332° = \sin(360° - 28°) = -\sin 28°$
15. ✗ $\cos 245° = \cos(180° + 65°) = -\cos 65°$;
    $-\cos 205° = -\cos(180° + 25°) = -(-\cos 25°) = \cos 25°$
16. ✓ $\tan 111° = \tan(180° - 69°) = -\tan 69°$

17. ✗ $\tan 255° = \tan(180° + 75°) = \tan 75°$; $\tan 165° = \tan(180° - 15°) = -\tan 15°$
18. ✓ $\sin 260° = \sin(180° + 80°) = -\sin 80°$; $\sin 280° = \sin(360° - 80°) = -\sin 80°$

2-4

1. (a) $\sin\theta = \sin(180° - 104°) = \sin 76°$, $\theta = 76°$ or 或 $\theta = 104°$
   (b) $\theta = 180° + 14° = 194°$ or 或 $360° - 14° = 346°$
   (c) 215°, 325° (d) 265°, 275° (e) 248°, 292°
   (f) 32°, 148° (g) 227°, 313° (h) 44°, 136°
2. (a) 75°, 285° (b) 162°, 198° (c) 64°, 296°
   (d) 150°, 210° (e) 12°, 348° (f) 73°, 287°
   (g) 23°, 337° (h) 179°, 181°
3. (a) 44°, 224° (b) 111°, 291° (c) 51°, 231°
   (d) 171°, 351° (e) 150°, 330° (f) 164°, 344°
   (g) 136°, 316° (h) 153°, 333°
4. (a) $x = 35.3°$ or 或 $360° - 35.3° = 324.7°$
   (b) $x = 61.5°$ or 或 $180° - 61.5° = 118.5°$
   (c) 19.4°, 160.6° (d) 44.5°, 224.5° (e) 81.2°, 278.8°
   (f) 87.9°, 92.1° (g) 72.2°, 252.2° (h) 4.9°, 355.1°
5. (a) $\theta = 180° + 85.6° = 265.6°$ or 或 $360° - 85.6° = 274.4°$
   (b) 114.4°, 294.4° (c) 119.1°, 299.1° (d) 101.6°, 258.4°
   (e) 152.7°, 207.3° (f) 258.2°, 281.8° (g) 112.5°, 292.5°
   (h) 130.1°, 229.9°

2-5

1. (a) $\frac{\sqrt{6}}{2}$ (b) 1 (c) $\frac{3}{4}$ (d) $\frac{\sqrt{6}}{6}$
   (e) $\frac{1}{2}$ (f) $\frac{\sqrt{6}}{3}$ (g) 9 (h) 1
2. (a) $= (\frac{1}{2})^2(1) - (\frac{\sqrt{3}}{2})(\frac{1}{\sqrt{3}})^2 = \frac{3-2\sqrt{3}}{12}$
   (b) $= \dfrac{(\frac{\sqrt{2}}{2})(\sqrt{3}) + (\frac{\sqrt{3}}{2})(\frac{1}{2})}{\frac{1}{\sqrt{3}}} = \frac{6\sqrt{2}+3}{4}$
   (c) $= \dfrac{\frac{1}{\sqrt{3}} + \sqrt{3}}{2} = \frac{2\sqrt{3}}{3}$
   (d) $= \dfrac{\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{\sqrt{2}}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}} = \frac{1+\sqrt{2}+\sqrt{3}}{1+\sqrt{3}-\sqrt{2}}$
   (e) $= (\frac{1}{2})(\frac{1}{2})^3 + (\frac{\sqrt{3}}{2})(\frac{\sqrt{3}}{2})^3 = \frac{5}{8}$
   (f) $= 4(\frac{\sqrt{3}}{2})^3 - 3(\frac{\sqrt{3}}{2}) = 0$

(g) $= \dfrac{(\frac{1}{\sqrt{3}})^2 - (\frac{1}{2})^2}{(\sqrt{3})^2 + (\frac{1}{2})^2} = \dfrac{1}{39}$

(h) $= \dfrac{1+\frac{\sqrt{3}}{2}+\frac{1}{2}}{1-\frac{1}{2}-\frac{\sqrt{3}}{2}} = \dfrac{3+\sqrt{3}}{1-\sqrt{3}} = \dfrac{\sqrt{3}(1+\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})} = \dfrac{\sqrt{3}(4+2\sqrt{3})}{-2} = -3-2\sqrt{3}$

2-6

1. (a) $\frac{\sqrt{3}}{2}, -\frac{1}{2}, -\sqrt{3}$ (b) $\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -1$ (c) $\frac{1}{2}, -\frac{\sqrt{3}}{2}, -\frac{\sqrt{13}}{3}$
   (d) $-\frac{1}{2}, -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{3}$ (e) $-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 1$ (f) $-\frac{\sqrt{3}}{2}, -\frac{1}{2}, \sqrt{3}$
   (g) $-\frac{\sqrt{3}}{2}, \frac{1}{2}, -\sqrt{3}$ (h) $-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -1$ (i) $-\frac{1}{2}, \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{3}$

2. (a) $= \dfrac{(-\sqrt{3})}{(-\frac{1}{2})} + \dfrac{(-\frac{1}{2})}{(-1)} - \dfrac{(\frac{\sqrt{3}}{2})}{(-\frac{\sqrt{3}}{2})} = \dfrac{3+4\sqrt{3}}{2}$ (b) $= \dfrac{\frac{1}{2}\cdot\frac{1}{2}+(-\frac{1}{2})\cdot(-\frac{1}{2})}{(-\frac{1}{2})(-\frac{\sqrt{3}}{2})+(-\frac{\sqrt{3}}{2})(-\frac{1}{2})} = \dfrac{\sqrt{3}}{3}$
   (c) $(\frac{\sqrt{3}}{2})^2(-\frac{1}{2}) - (\frac{1}{2})(\frac{1}{2})^2 = -\frac{1}{2}$ (d) $(-1)^3(-\frac{\sqrt{2}}{2})^2(-\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{4}$

3. (a) F (b) F (c) F (d) T (e) T (f) F (g) T (h) F

4. (a) 0°, 180°, 360° (b) 0°, 180°, 360° (c) 90°, 270°
   (d) 180° (e) 90° (f) 0°, 360°
   (g) 270° (h) 45°, 225°

2-7

1. (a) 30°, 150° (b) 135°, 225° (c) 120°, 300° (d) 30°, 330°
2. (a) 19.5°, 160.5° (b) 242.7°, 297.3° (c) 187.2°, 352.8°
   (d) $\sin\theta = \frac{7}{3} > 1$, ∴ no solution 無解
3. (a) 106.6°, 253.4° (b) 56.3°, 303.7°
   (c) $\cos\theta = \frac{8}{3} > 1$, ∴ no solution 無解 (d) 149.0°, 211.0°
4. (a) 53.1°, 233.1° (b) 102.5°, 282.5° (c) 39.8°, 219.8°
   (d) 150.9°, 330.9°
5. (a) 41.8°, 138.2°, 221.8° or 或 318.2° (b) 42.8°, 137.2°, 222.8° or 或 317.2°
   (c) 32.3°, 147.7°, 212.3° or 或 327.7° (d) 68.2°, 111.8°, 248.2° or 或 291.8°

2-8

1. $0° \le 2x \le 720°$; $2x = 60°, 240°, 420°$ or 或 $600°$, $x = 30°, 120°, 210°$ or 或 $300°$
2. $0° \le 3x \le 1080°$; $3x = 20.4°, 159.6°, 380.4°, 519.6°, 740.4°$ or 或 $879.6°$,
   $x = 6.8°, 53.2°$, $126.8°, 173.2°, 246.8°$ or 或 $293.2°$
3. $0° \le \frac{x}{2} \le 180°$; $\frac{x}{2} = 40.1°$, $x = 80.2°$
4. $0° \le \frac{x}{2} \le 180°$; $\frac{x}{2} = 140.1°$, $x = 280.1°$
5. $0° \le \frac{x}{3} \le 120°$; $\frac{x}{3} = 57.1°$, $171.3°$
6. $0° \le 2x \le 720°$; $2x = 145°, 215°, 505°$ or 或 $575°$,
   $x = 72.5°, 107.5°, 252.5°$ or 或 $287.5°$
7. $50° \le x + 50° \le 410°$; $x + 50° = 140.2°$ or 或 $399.8°$, $x = 90.2°$ or 或 $349.8°$
8. $-15° \le x - 15° \le 345°$; $x - 15° = 120°$ or 或 $240°$, $x = 135°$ or 或 $255°$
9. $20° \le x + 20° \le 380°$; $x + 20° = 47.5°$ or 或 $227.5°$, $x = 27.5°$ or 或 $207.5°$
10. $30° \le 2x + 30° \le 750°$; $2x + 30° = 37.0°$, $143.0°$, $397.0°$ or 或 $503.0°$
    $x = 3.5°$, $56.5°$, $183.5°$ or 或 $236.5°$
11. $-25° \le \frac{x}{2} - 25° \le 155°$; $\frac{x}{2} - 25° = 56.4°$, $x = 162.8°$
12. $-10° \le \frac{x}{4} - 10° \le 80°$; $\frac{x}{4} - 10° = 35.4°$, $x = 181.5°$

2-9

1. (a) $\sin\theta$ (b) $1 + 2\sin\theta\cos\theta$ (c) $1 - 2\sin\theta\cos\theta$ (d) $\frac{1}{\cos^2\theta}$

   (e) $\cos^2\theta$ (f) $2\sin^2\theta$

   (g) $= \frac{(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)}{\cos^2\theta} = 1 - \tan^2\theta$ (h) $= \frac{1 - \sin^2\theta}{\cos^2\theta} = 1$

2. (a) L.H.S.左方 $= \frac{\sin\theta(\sin^2\theta + \cos^2\theta)}{\cos\theta} = \tan\theta =$ R.H.S.右方

   (b) L.H.S.左方 $= \frac{(1 - 2\sin\theta + \sin^2\theta) + \cos^2\theta}{\cos\theta(1 - \sin\theta)} = \frac{2(1 - \sin\theta)}{\cos\theta(1 - \sin\theta)} = \frac{2}{\cos\theta} =$ R.H.S.右方

   (c) L.H.S.左方 $= (\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta})\cos^2\theta = 1 =$ R.H.S.右方

   (d) L.H.S.左方 $= 1 + 2(\frac{\sin\theta}{\cos\theta})(\cos^2\theta) = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = (\sin\theta + \cos\theta)^2$

   = R.H.S 右方

   (e) L.H.S.左方 $= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \frac{1}{1 - \cos^2\theta} =$ R.H.S.右方

   (f) L.H.S.左方 $= \tan^2\theta(\tan\theta - 1) + (\tan\theta - 1) = (\tan^2\theta + 1)(\tan\theta - 1)$

   $= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta}(\tan\theta - 1) = \frac{\tan\theta - 1}{\cos^2\theta} =$ R.H.S.右方

(g) R.H.S.右方 $=\dfrac{1-\sin^2\theta}{1+\sin\theta}=\dfrac{(1-\sin\theta)(1+\sin\theta)}{1+\sin\theta}=1-\sin\theta=$ L.H.S.左方

(h) L.H.S.左方 $=\dfrac{\sin\theta(1+\sin\theta)-\sin\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}=\dfrac{2\sin^2\theta}{1-\sin^2\theta}=\dfrac{2\sin^2\theta}{\cos^2\theta}=2\tan^2\theta$

$=$ R.H.S 右方

2-10

1. $(3\tan\theta-1)(5\tan\theta+6)=0$，$\tan\theta=\dfrac{1}{3}$ or 或 $-\dfrac{6}{5}$(rejected 捨去),
   $\theta=18.4°$, 129.8°, 198.4° or 或 309.8°
2. $(2\cos\theta+1)(\cos+1)=0$，$\cos\theta=-\dfrac{1}{2}$ or 或 -1，$\theta=120°$，180° or 或 240°
3. $\cos\theta(\cos\theta-1)=0$，$\cos\theta=0$ or 或 1，$\theta=0°$，90°，270° or 或 360°
4. $(3\sin\theta+2)(2\sin\theta-1)=0$，$\sin\theta=-\dfrac{2}{3}$ or 或 $\dfrac{1}{2}$，$\theta=30°$，150°，221.8° or 或 318.2°
5. $(7\cos\theta-1)(4\cos\theta-3)=0$，$\cos\theta=\dfrac{1}{7}$ or 或 $\dfrac{3}{4}$，$\theta=41.4°$，81.8°，278.2° or 或 318.6°
6. $\sin\theta(2\sin\theta+1)=0$，$\sin\theta=0$ or 或 $-\dfrac{1}{2}$，$\theta=0°$，180°，210°，330° or 或 360°
7. $\tan\theta(5\tan\theta-2)=0$，$\tan\theta=0$ or 或 $\dfrac{2}{5}$，$\theta=0°$，21.8°，180°，201.8° or 或 360°
8. $(5\tan\theta-2)(3\tan\theta+2)=0$，$\tan\theta=\dfrac{2}{5}$ or 或 $-\dfrac{2}{3}$，$\theta=21.8°$,146.3°,201.8° or 或 326.3°
9. $(5\sin\theta-4)(2\sin\theta-1)=0$，$\sin\theta=\dfrac{4}{5}$ or 或 $\dfrac{1}{2}$，$\theta=30°$，53.1°，126.9° or 或 150°
10. $(\tan\theta-1)(3\tan\theta+8)=0$，$\tan\theta=1$ or 或 $-\dfrac{8}{3}$，$\theta=45°$，110.6°，225° or 或 290.6°
11. $\tan\theta=7.40512$ or 或 -0.40512，$\theta=82.3°$，157.9°，262.3° or 或 337.9°
12. $\sin\theta=0.84564$ or 或 -0.29564，$\theta=57.7°$，122.3°，197.2° or 或 342.8°
13. $\tan\theta=-0.05418$ or 或 -1.84582，$\theta=118.4°$，176.9°，298.4° or 或 356.9°
14. $\cos\theta=0.52190$ or 或 -0.10085，$\theta=58.5°$，95.8°，264.2° or 或 301.5°

2-11

1. $4\sin^2x+15\sin x+3=0$，$\sin x=-0.21198$ or 或 -3.53802 (rejected 捨去),
   $x=192.2°$ or 或 347.8°
2. $3\cos^2x-7\cos x-7=0$，$\cos x=-0.75543$ or 或 3.08876 (rejected 捨去),
   $x=139.1°$ or 或 220.9°
3. $2\cos^2x+5\cos x+1=0$，$\cos x=-0.21922$ or 或 -2.28078 (rejected 捨去),
   $x=102.7°$ or 或 257.3°

4. $5\sin^2 x - 7\sin x + 1 = 0$, $\sin x = 0.16148$ or 或 1.23852 (rejected 捨去),
$x = 9.3°$ or 或 $170.7°$

5. $9\sin^2 x + 2\sin x - 9 = 0$, $\sin x = 0.89504$ or 或 -1.11727 (rejected 捨去),
$x = 63.5°$ or 或 $116.5°$

6. $3\cos^2 x + 5\cos x - 1 = 0$, $\cos x = 0.18046$ or 或 -1.84713 (rejected 捨去),
$x = 79.6°$ or 或 $280.4°$

7. $11\sin^2 x - 12\sin x + 1 = 0$, $\sin x = 1$ or 或 $\frac{1}{11}$, $x = 5.2°$, $90°$ or 或 $174.8°$

8. $7\sin^2 x - 8\sin x - 7 = 0$, $\sin x = -0.58032$ or 或 1.72318 (rejected 捨去),
$x = 215.5°$ or 或 $324.5°$

9. $6\sin^2 x - \sin x - 4 = 0$, $\sin x = 0.90407$ or 或 -0.73740,
$x = 64.7°, 115.3°, 227.5°$ or 或 $312.5°$

10. $2\cos^2 x - 3\cos x - 4 = 0$, $\cos x = -0.85078$ or 或 2.35078 (rejected 捨去),
$x = 148.3°$ or 或 $211.7°$

11. $9\cos^2\theta - 3\cos\theta - 2 = 0$, $\cos\theta = -\frac{1}{3}$ or 或 $\frac{2}{3}$,
$\theta = 48.2°$, $109.5°$, $250.5°$ or 或 $311.8°$

12. $\cos^2\theta - 7\cos\theta + 5 = 0$, $\cos\theta = 0.80742$ or 或 6.19258 (rejected 捨去),
$\theta = 36.2°$ or 或 $323.8°$

13. $7\cos^2\theta - 4\cos\theta - 2 = 0$, $\cos\theta = 0.89181$ or 或 -0.32038,
$\theta = 26.9°$, $108.7°$, $251.3°$ or 或 $333.1°$

14. $\cos^2\theta + 5\cos\theta - 2 = 0$, $\cos\theta = 0.37228$ or 或 -5.37228 (rejected 捨去),
$\theta = 68.1°$ or 或 $291.9°$

2-12

1. $\tan\theta = \frac{9}{2}$, $\theta = 77.5°$, $257.5°$

2. $\tan\theta = -\sqrt{3}$, $\theta = 120°$, $300°$

3. $\tan\theta = -2$, $\theta = 116.6°$, $296.6°$

4. $\tan\theta = \frac{1}{6}$, $\theta = 9.5°$, $189.5°$

5. $\tan\theta = \frac{8}{3}$, $\theta = 69.4°$, $249.4°$

6. $\tan\theta = -7$, $\theta = 98.1°$, $278.1°$

7. $15\tan^2\theta - 22\tan\theta + 8 = 0$, $\tan\theta = \frac{2}{3}$ or 或 $\frac{4}{5}$, $\theta = 33.7°$, $38.7°$, $213.7°$ or 或 $218.7°$

8. $6\tan^2\theta - 13\tan\theta - 5 = 0$, $\tan\theta = -\frac{1}{3}$ or 或 $\frac{5}{2}$, $\theta = 68.2°$, $161.6°$, $248.2°$ or 或 $341.6°$

9. $6\tan^2\theta - 31\tan\theta + 5 = 0$, $\tan\theta = \frac{1}{6}$ or 或 5, $\theta = 9.5°$, $78.7°$, $189.5°$ or 或 $258.7°$

10. $30\tan^2\theta + \tan\theta - 3 = 0$, $\tan\theta = -\frac{1}{3}$ or 或 $\frac{3}{10}$, $\theta = 16.7°$, $161.6°$, $196.7°$ or 或 $341.6°$

11. $7\tan^2\theta+2\tan\theta-9=0$, $\tan\theta=-\frac{9}{7}$ or 或 1, $\theta=45°$, $127.9°$, $225°$ or 或 $307.9°$

12. $3\tan^2\theta+\tan\theta-24=0$, $\tan\theta=-3$ or 或 $\frac{8}{3}$, $\theta=69.4°$, $108.4°$, $249.4°$ or 或 $288.4°$

13. $8\sin^2\theta-5\sin\theta\cos\theta-8\cos^2\theta=3\sin^2\theta+3\cos^2\theta$, $5\tan^2\theta-5\tan\theta-11=0$,
$\tan\theta=\frac{5\pm7\sqrt{5}}{10}$, $\theta=64.2°$, $133.2°$, $244.2°$ or 或 $313.2°$

14. $\sin^2\theta+6\sin\theta\cos\theta+\cos^2\theta=-\sin^2\theta-\cos^2\theta$, $2\tan^2\theta+6\tan\theta+2=0$,
$\tan\theta=\frac{-3\pm\sqrt{5}}{2}$, $\theta=110.9°$, $159.1°$, $290.9°$ or 或 $339.1°$

15. $3\sin^2\theta+2\sin\theta\cos\theta-3\cos^2\theta=2\sin^2\theta+2\cos^2\theta$, $\tan^2\theta+2\tan\theta-5=0$,
$\tan\theta=-1\pm\sqrt{6}$, $\theta=55.4°$, $106.2°$, $235.4°$ or 或 $286.2°$

16. $8\sin^2\theta+4\sin\theta\cos\theta=\sin^2\theta+\cos^2\theta$, $7\tan^2\theta+4\tan\theta-1=0$,
$\tan\theta=\frac{-2\pm\sqrt{11}}{7}$ or 或 $\frac{8}{3}$, $\theta=10.7°$, $142.8°$, $190.7°$ or 或 $322.8°$

2-13

1. (a) $\tan x=\frac{7}{8}$, $x=41.2°$ or 或 $221.2°$

(b) $\cos x=\pm\frac{2}{3}$, $x=48.2°$, $131.8°$, $228.2°$ or 或 $311.8°$

(c) $\tan x=\pm\frac{\sqrt{5}}{2}$, $x=48.2°$, $131.8°$, $228.2°$ or 或 $311.8°$

(d) $\tan x=-\frac{1}{2}$, $x=153.4°$ or 或 $333.4°$

(e) $\sin\frac{x}{3}=1$, $\frac{x}{3}=90°$, $x=270°$

(f) $2x+75°=90°$, $270°$, $450°$ or 或 $630°$, $x=7.5°$, $97.5°$, $187.5°$ or 或 $277.5°$

(g) $\tan x=0$ or 或 $\tan x=-\frac{1}{4}$, $x=0°$, $166.0°$, $180°$, $346.0°$ or 或 $360°$

(h) $\cos x=-\frac{1}{2}$ or 或 $\cos x=1$, $x=0°$, $120°$, $240°$ or 或 $360°$

2. (a) $5\sin^2x-3\sin x-3=0$, $\sin x=-0.53066$ or 或 1.13066 (rejected 捨去),
$x=212.1°$ or 或 $327.9°$

(b) $\sin x=-0.41421$ or 或 2.41421 (rejected 捨去), $x=204.5°$ or 或 $335.5°$

(c) $15\cos^2x-11\cos x+2=0$, $\cos x=\frac{2}{5}$ or 或 $\frac{1}{3}$,
$x=66.4°$, $70.5°$, $289.5°$ or 或 $293.6°$

(d) $\tan x=2$ or 或 $\tan x=-1$, $x=63.4°$, $135°$, $243.4°$ or 或 $315°$

(e) $\sin^2 x-\sin x\cos x-4\cos^2 x=0$，$\tan x=2.56155$ or 或 -1.56155，
$x=68.7°$，$122.6°$，$248.7°$ or 或 $302.6°$

(f) $6\sin^2 x-\sin x\cos x-\cos^2 x=0$，$\tan x=\frac{1}{2}$ or 或 $-\frac{1}{3}$，
$x=26.6°$，$161.6°$，$206.6°$ or 或 $341.6°$

(g) $15\sin^2 x+7\sin x-4=0$，$(1-\cos^2\theta)(\frac{1}{\tan^2\theta})=\cos^2\theta$ or 或 $\cos^2\theta+\sin^2\theta=1$，
$x=19.5°$，$160.5°$，$233.1°$ or 或 $306.9°$

(h) $40\sin^2 x-41\sin x\cos x+10\cos^2 x=0$，$\tan x=\frac{2}{5}$ or 或 $\frac{\cos\theta}{-\sin\theta}\cdot(-\tan\theta)=1$，
$x=21.8°$，$32.0°$，$201.8°$ or 或 $212.0°$

(i) $(2\tan x-3)(\tan^2 x+1)=0$， $\therefore\ \tan x=\frac{3}{2}$，$x=56.3°$ or 或 $236.3°$

2-14

1. $\sin\theta+\frac{1}{\tan\theta}\cdot\cos\theta=\frac{1}{\sin\theta}$
2. $\tan\theta\cdot\frac{1}{\tan\theta}-\sin^2\theta=\cos^2\theta$
3. $\frac{\sin^2\theta}{\cos\theta}\cdot\frac{1}{\tan\theta}=\sin\theta$
4. $1+\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\tan\theta}=\frac{1}{\sin^2\theta}$
5. $\frac{1-\sin^2\theta}{\cos\theta}=\cos\theta$
6. $\frac{\sin^2\theta}{\cos^2\theta}-\frac{1}{\cos^2\theta}=-1$
7. $\tan\theta\cos\theta+\sin\theta=2\sin\theta$
8. $\frac{1}{\tan\theta}\cdot\sin\theta=\cos\theta$
9. $\frac{\sin\theta(\sin\theta+\cos\theta)}{\sin\theta\cos\theta+\cos^2\theta}=\tan\theta$
10. $\frac{\sin\theta\cos\theta}{1-\sin^2\theta}=\tan\theta$
11. $\frac{\sin 50°}{\sin(90°-40°)}=1$
12. $\frac{\sin^2 52°}{\cos^2 52°}\cdot\cos(90°-38°)+\cos^2(90°-38°)=1$
13. $1-\tan 76°\cdot\tan(90°-14°)=0$
14. $\cos(90°-26°)\cdot\tan 64°\cdot\sin(90°-26°)+\cos^2 64°=1$
15. $\frac{\cos(90°-61°)\cdot\tan 29°}{\cos 29°}\cdot\tan(90°-29°)=1$
16. $\tan 47°\cdot\cos(90°-43°)-\sin(90°-43°)=0$
17. $(\tan 1°\cdot\tan 89°)(\tan 2°\cdot\tan 88°).....(\tan 44°\cdot\tan 46°)\tan 45° = (1)(1)....(1)\tan 45° = 1$
18. $(\cos^2 1°+\cos^2 89°)+(\cos^2 3°+\cos^2 87°)+....+(\cos^2 43°+\cos^2 47°)+\cos^2 45°$

$= (\cos^2 1°+\sin^2 1°)+(\cos^2 3°+\sin^2 3°)+....+(\cos^2 43°+\sin^2 43°)+(\frac{1}{\sqrt{2}})^2$

$= 1\times 22+\frac{1}{2} = 22.5$

2-15

1. $(1-\cos^2\theta)(\frac{1}{\tan^2\theta})=\cos^2\theta$
2. $\cos^2\theta+\sin^2\theta=1$
3. $1-\cos^2\theta=\sin^2\theta$
4. $(-\tan\theta)(\cos\theta)=-\sin\theta$
5. $1+\frac{\cos\theta}{\tan\theta}\cdot\sin\theta=1+\cos^2\theta$
6. $\frac{\cos\theta}{-\sin\theta}\cdot(-\tan\theta)=1$
7. $\frac{-\cos\theta}{-\sin\theta}-\frac{1}{\tan\theta}=0$
8. $(1+\frac{\cos^2\theta}{1-\sin^2\theta})(-\sin\theta)(-\cos\theta)=2\sin\theta\cos\theta$
9. $\frac{\sin\theta}{\sin\theta}-(\frac{1}{\cos^2\theta})+\frac{\sin^2\theta}{1-\cos^2\theta}=2-\frac{1}{\cos^2\theta}$
10. $\frac{\sin\theta}{-\sin\theta}\cdot\frac{\frac{-1}{\tan\theta}\cdot(-\sin\theta)}{\cos\theta}=-1$
11. $\text{L.H.S.}=\frac{1}{\cos^2\theta}-1=\tan^2\theta=\text{R.H.S.}$
12. $\text{L.H.S.}=\frac{\cos^2\theta}{\sin^2\theta}+1=\frac{1}{\sin^2\theta}=\text{R.H.S.}$
13. $\text{R.H.S.}=(1-\cos\theta)(1+\cos\theta)=\sin^2\theta=\text{L.H.S.}$
14. $\text{L.H.S.}=(\frac{\sin\theta}{\cos\theta}+\tan\theta\cdot\frac{1}{\tan\theta})^2-(\frac{\sin\theta}{\cos\theta}-\frac{\tan\theta}{\tan\theta})^2$

    $=\tan^2\theta+2\tan\theta+1-\tan^2\theta+2\tan\theta-1=4\tan\theta=\text{R.H.S.}$

2-16

1. $\theta=37°$ or 或 $323°$
2. $\theta=234°$ or 或 $306°$
3. $\theta=32°$ or 或 $148°$
4. $\theta=28°$ or 或 $332°$
5. $\theta=145°$ or 或 $215°$
6. $\theta=81°$ or 或 $99°$
7. $\theta=144°$ or 或 $324°$
8. $\theta=178°$ or 或 $182°$
9. $\theta=22°$ or 或 $202°$
10. $\theta=14°$ or 或 $194°$
11. $\theta=50°$ or 或 $230°$
12. $\theta=48°$ or 或 $132°$

2-17

1. (a) $f(90)=2$, $f(-90)=0$, $f(56)=1.83$
   (b) $f(120)=5$, $f(-150)=6.46$, $f(288)=1.76$
   (c) $f(150)$ undefined 未下定義, $f(-225)=3.73$, $f(315)=3.73$
   (d) $f(115)=7.37$, $f(-75)=1.14$, $f(218)=6.27$
   (e) $f(25)=0.863$, $f(-235)=1.67$, $f(335)=0.950$
2. (a) $\tan x=\frac{5}{2}$, $x=68.2°$ or 或 $248.2°$ (b) $\sin\theta=\frac{1}{2}$, $\theta=30°$ or 或 $150°$
   (c) $\cos y=\pm1$, $y=0°$, $180°$ or 或 $360°$
3. (a) $-a+5=2$, $a=3$ (b) $-\frac{b}{2}-8=-1$, $b=-14$ (c) $-2+c=13$, $c=15$

2-18

1. (a) $3 \le 8 - 5\sin x \le 13$ (b) $\frac{15}{2} \le 9 + \frac{3\cos x}{2} \le \frac{21}{2}$ (c) $\frac{1}{3} \le \frac{1}{5 + 2\cos x} \le \frac{1}{7}$

   (d) $\frac{3}{5} \le \frac{3}{4 - \sin x} \le 1$ (e) $-2 \le 3\sin^2 x - 2 \le 1$ (f) $-\frac{1}{3} \le \frac{1}{\cos^2 x - 4} \le -\frac{1}{4}$

2. (a) $-2 \le 1 - 3\cos 2x \le 4$ (b) $-9 \le 2\sin 3x - 7 \le -5$

   (c) $1 \le 4 - 3\sin(x - 30°) \le 7$ (d) $-6 \le 6\cos(4x - 110°) \le 6$

   (e) $5 \le 5 + 4\cos^2(x + 40°) \le 9$ (f) $1 \le (2\sin x - 3)^2 \le 25$

3. (a) $0 \le 1 - \sin\theta \le 1$ (b) $-7 \le 3\cos 2\theta - 4 \le -1$

   (c) $2 \le 2 + \cos(2\theta - 90°) \le 3$ (d) $-4 \le 4\sin(\frac{\theta + 180°}{2}) \le 2\sqrt{2}$

4. (a) $3 \le (\sin x - 3)^2 - 1 \le 15$ (b) $5 \le 3(\cos x + 1)^2 + 5 \le 17$

2-19

1. (a)

x	0°	45°	90°	135°	180°	225°	270°	315°	360°
y	0	1	0	−1	0	1	0	−1	0

   (b)

x	−60°	30°	120°	210°	300°	390°
y	0	1	0	−1	0	1

   (c)

x	0°	90°	180°	270°	360°	450°	540°	630°	720°
y	0	1	U	−1	0	1	U	−1	0

   Remark: "U" means undefined. 備註："U"表示未下定論

2. (a) $b = \frac{\text{max.極大值} + \text{min.極小值}}{2} = -2$, $a = \text{max.極大值} - b = 2$,

   period 周期 $= 300° - (-60°) = 360°$, $\therefore m = 1$, $n = 60°$

   (b) $b = 1$, $a = -3$, period 周期 $= 195° - 15° = 180°$, $\therefore m = 2$,

   sub. 代 $(60°, -2)$ into 入 $y = -3\sin(2x + n) + 1$, $n = -30°$

   (c) $b = -9$, $a = -8$, period 周期 $= 210° - 30° = 180°$, $\therefore m = 2$, $n = 45°$

3. (a) $b = \frac{\text{max.極大值} + \text{min.極小值}}{2} = 7$, $a = \text{max.極大值} - b = 4$, $\therefore m = 1$, $n = -120°$

   (b) $b = -1$, $a = 2$, period 周期 $= 660° - (-60°) = 720°$, $\therefore m = \frac{1}{2}$, $n = 30°$

   (c) $b = 8$, $a = 1$, period 周期 $= 130° - 10° = 120°$, $\therefore m = 3$, $n = -30°$

4. (a) centre of rotational at 旋轉中心位於 $(45°, 1)$, $\therefore b = 1$, $n = -45°$

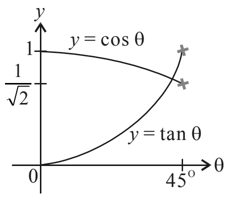
   period 周期 $= 180°$ $\therefore m = 1$, sub.代 $(0°, 2)$ into 入 $y = a\tan(x - 45°) + 1$, $a = -1$

   (b) centre of rotational at 旋轉中心位於 $(45°, 0)$, $\therefore b = 0$, $n = -45°$

   period 周期 $= 180°$ $\therefore m = 1$, sub.代 $(0°, -3)$ into 入 $y = a\tan(x - 45°)$, $a = 3$

   (c) centre of rotational at 旋轉中心位於 $(90°, 7)$, $\therefore b = 7$, $n = -90°$

   period 周期 $= 180°$ $\therefore m = 1$, sub.代 $(45°, 6)$ into 入 $y = a\tan(x - 90°) + 7$, $a = 1$

5. 

max. 極大值 $= \tan 45° - \cos 45° = 1 - \frac{1}{\sqrt{2}}$

min. 極小值 $= \tan 0° - \cos 0° = -1$

2-20

1. $QP : QR = 3 : 5$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{\sqrt{5^2 - 3^2}}{3} = \frac{4}{3}$

2. $\tan\theta < 0$ ($\because 4^{th}$ quadrant 第四象限), $\tan\theta = \frac{-\sqrt{k^2-1}}{1}$,

   $\therefore \tan(\theta - 270°) = -\frac{1}{\tan\theta} = \frac{1}{\sqrt{k^2-1}}$

3. $\cos(180° + \theta) - \cos(90° - \theta) = -\cos\theta - \sin\theta = -\frac{5}{13} - \frac{\sqrt{13^2 - 5^2}}{13} = -\frac{17}{13}$

4. (a) $\frac{\sin^2\theta}{\cos^2\theta} = 1$, $\tan^2\theta = 1$, $\tan\theta = \pm 1$, $\theta = 45°$, $135°$, $225°$, $315°$

   (b) $2\sin 2\theta - 1 = 0$ or 或 $\cos\theta + 1 = 0$, $\sin 2\theta = \frac{1}{2}$ or 或 $\cos\theta = -1$;

   $\therefore \theta = 15°$, $75°$, $195°$, $255°$ or 或 $180°$
   ($\because 0° \le 2\theta \le 720°$, $2\theta = 30°$, $150°$, $390°$ or 或 $510°$)

5. $(3\tan\theta + 1)(\tan\theta - 1) = 0$, $\tan\theta = -\frac{1}{3}$ or 或 1

6. $\cos^2 x = \frac{1}{4}$, $\cos x = \pm\frac{1}{2}$, $x = 240°$ or 或 $300°$

7. $2\cos^2\theta - 3\cos\theta + 1 = 0$, $(2\cos\theta - 1)(\cos\theta - 1) = 0$,

   $\cos\theta = \frac{1}{2}$ or 或 1, $\theta = 0°$ or 或 $60°$ ($\because 0° \le \theta \le 90°$)

8. (a) $= 1 - 2(1) = -1$ (b) $= 0$ (c) $= \frac{3}{1+2} = 1$

9. (a) $= 3 + 4(1) = 7$ (b) $= \frac{1}{3^{1-1}} = 1$ (c) $= [2(-1) - 1]^2 + 3 = 12$

   (d) $4\cos^2 x + 3(1 - \cos^2 x) - 1 = \cos^2 x + 2$, $\therefore$ max.極大值 $= (1)^2 + 2 = 3$

10. (a) $= \frac{\cos\theta}{-\cos\theta} = -1$

   (b) $= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta) + 2\cos^2\theta = \sin^2\theta - \cos^2\theta + 2\cos^2\theta = 1$

   (c) $= \sin^2\theta(1 - \sin^2\phi) - \sin^2\phi(1 - \sin^2\theta) = \sin^2\theta - \sin^2\phi$

   (d) $= (1 - \sin\theta)(1 + \sin\theta) = 1 - \sin^2\theta = \cos^2\theta$

   (e) $= \frac{1 - \cos^2\theta}{1 + \cos\theta} - 1 = \frac{(1 + \cos\theta)(1 - \cos\theta)}{1 + \cos\theta} - 1 = (1 - \cos\theta) - 1 = -\cos\theta$

   (f) $= (\frac{1 - \cos\theta}{\sin\theta})(1 + \cos\theta) = \frac{1 - \cos^2\theta}{\sin\theta} = \sin\theta$

11. $=1+\cos A\cos(180°-A)=1-\cos^2 A=\sin^2 A$

12. $(\cos\theta-\sin\theta)^2=\frac{4}{9}$, $\cos^2\theta-2\cos\theta\sin\theta+\sin^2\theta=\frac{4}{9}$, $1-2\cos\theta\sin\theta=\frac{4}{9}$, $\cos\theta\sin\theta=\frac{5}{18}$

13. (a) $=\sin^2 1°+\cos^2 1°=1$

(b) $=(\sin^2 1°+\sin^2 89°)+(\sin^2 2°+\sin^2 88°)+...+(\sin^2 44°+\sin^2 46°)+\sin^2 45°$

$=1\times 44+(\frac{1}{\sqrt{2}})^2=44.5$

3-1

1. (a) 9.6 cm^2 (b) 39.0 cm^2 (c) 21.2 cm^2
2. (a) 250 (b) 262.9 (c) 144.3
3. (a) $\frac{1}{2}(x)(30)\sin 150°=120$, $x=16$ (b) $2\times\frac{1}{2}(8)(y)\sin 80°=118.13$, $y=15.0$

(c) $\frac{1}{2}(22)(z)\sin 35°=227$, $z=36.0$

4. (a) $\frac{1}{2}(62)(83)\sin\theta=1619$, $\theta=39.0°$ or 或 $141.0°$

(b) $\frac{1}{2}(21)(46)\sin\theta=227$, $\theta=28.0°$ or 或 $152.0°$

(c) $\frac{1}{2}(\frac{56}{10})(\frac{34}{10})\sin\theta=9.52$, $\theta=90.0°$

5. (a) $\angle DOC=360°\div 5=72°$, $5(\frac{1}{2})(17)(17)\sin 72°=687.1\text{ cm}^2$

(b) $\angle QOR=360°\div 8=45°$, $8(\frac{1}{2})(8)(8)\sin 45°=181.0\text{ cm}^2$

3-2

In this exercise, s = (perimeter 周界) ÷ 2, A = area of triangle 三角形面積.

1. $s=112$, $A=1833.0\text{ mm}^2$ 2. $s=33$, $A=188.0\text{ cm}^2$
3. $s=88.5$, $A=770.8\text{ cm}^2$
4. $WE=\sqrt{16^2+12^2}=20$, $s=\frac{20+18+6}{2}=22$, $\therefore A=\frac{1}{2}(16)(12)+\sqrt{22(2)(4)(16)}=149.1\text{ cm}^2$
5. $SQ=\sqrt{5^2+12^2}=13$, $s=\frac{7+9+13}{2}=14.5$,

$\therefore A=\frac{1}{2}(5)(12)+\sqrt{14.5(1.5)(7.5)(5.5)}=60.0\text{ mm}^2$

6. $OT=OU=OV=25$, $\therefore A=\sqrt{32(7)(7)(18)}+\sqrt{45(20)(20)(5)}=168+300=468.0\text{ cm}^2$
7. $s=58.5$, $A=647.80$, $\frac{42\times h}{2}=647.80$, $h=30.8$
8. $s=18$, $A=32.86$, $\frac{13\times h}{2}=32.86$, $h=5.1$
9. $s=20$, $A=69.282$, $\frac{16\times h}{2}=69.28$, $h=8.7$ cm
10. $s=10$, $A=11.83$, $\frac{3\times h}{2}=11.83$, $h=7.9$ cm

3-3

1. (a) $\angle A = 75°$, $\dfrac{AC}{\sin 63°} = \dfrac{8}{\sin 42°}$, $AC = 10.7$ cm ; $\dfrac{BC}{\sin 75°} = \dfrac{8}{\sin 42°}$, $BC = 11.6$ cm

   (b) $\angle F = 30°$, $\dfrac{DF}{\sin 20°} = \dfrac{16}{\sin 130°}$, $DF = 7.1$ cm ; $\dfrac{DE}{\sin 30°} = \dfrac{16}{\sin 130°}$, $DE = 10.4$ cm

   (c) $\angle I = 25.4°$, $\dfrac{1.4}{\sin 25.4°} = \dfrac{GI}{\sin 82.3°}$, $GI = 3.2$ cm ; $\dfrac{1.4}{\sin 25.4°} = \dfrac{HI}{\sin 72.3°}$, $HI = 3.1$ cm

2. (a) $\dfrac{x}{\sin(110° - 22°)} = \dfrac{4}{\sin 22°}$, $x = 10.7$

   (b) $\sin 68° = \dfrac{9}{y}$, $y = 9.7$; $\dfrac{x}{\sin(68° - 35°)} = \dfrac{y}{\sin 35°}$, $x = 9.2$

   (c) $\theta = 106°$; $\dfrac{x}{\sin 106°} = \dfrac{23}{\sin 46°}$, $x = 30.7$; $\cos 54° = \dfrac{y}{x}$, $y = 18.1$

3. (a) $\angle R = 40°$, $\dfrac{x}{\sin 40°} = \dfrac{36}{\sin 19°}$, $x = 71.1$; $A = 1096.7\ \text{cm}^2$

   (b) $\angle P = 45°$, $\dfrac{x}{\sin 57°} = \dfrac{65}{\sin 45°}$, $x = 77.1$, $A = 2450.8\ \text{m}^2$

3-4

1. (a) $\sin\theta = 0.782$, $\theta = 51.5°$ or 或 $129°$
   (b) $\sin\theta = 0.392$, $\theta = 23.1°$ (reject 捨去 157°)
   (c) $\sin\theta = 1.286 > 1$, $\therefore$ no solution 沒有解
   (d) $\sin\theta = 0.951$, $\theta = 72.0°$ or 或 $\theta = 108°$
   (e) $\sin\theta = 0.217$, $\theta = 12.5°$ (reject 捨去 $167.5°$)
   (f) $\sin\theta = 0.868$, $\theta = 60.3°$ or 或 $118°$

2. $\dfrac{32}{\sin 24°} = \dfrac{40}{\sin L}$, $\sin L = 0.508$, $\angle L = 30.6°$ or 或 $149.4°$
   when 當 $\angle L = 30.6°$, $\angle K = 125.4°$, $JL = 64.1$ cm ;
   when 當 $\angle L = 149.4°$, $\angle K = 6.6°$, $JL = 9.0$ cm

3-5

1. (a) 4.8 (b) 22.7 (c) 5.7 (d) 34. 7 (e) 41.7 (f) 101.8
2. (a) $\cos\theta = 0.84$, $\theta = 32.8°$ (b) $\cos\theta = -0.6$, $\theta = 126.9°$
   (c) $\cos\theta = -0.089$, $\theta = 95.1°$
3. $KJ^2 = 42^2 + 39^2 - 2 \times 42 \times 39 \cos 34°$, KJ = 23.86 cm, ∠K = 66.1°, ∠J = 79.9°

3-6

1. $\dfrac{x}{\sin(70 - 40)°} = \dfrac{7}{\sin 40°}$, $x = 5.4$; $\dfrac{y}{\sin 70°} = \dfrac{7}{\sin 60°}$, $y = 7.6$

2. $HF^2 = 8^2 + 10^2 - 2(8)(10)\cos 65°$, $HF = 9.8$;
   $x^2 = 9^2 + HF^2 - 2(9)(HF)\cos 20°$, $x = 3.4$ cm

3. $\dfrac{8.6}{\sin 50°} = \dfrac{KI}{\sin 70°}$, $KI = 10.5$ cm ; $\sin 60° = \dfrac{x}{KI}$, $x = 9.1$ cm

4. $\dfrac{NP}{\sin 25^\circ}=\dfrac{37}{\sin(40-25)^\circ}$, $NP=60.4$ cm; $\sin 40^\circ=\dfrac{y}{NP}$, $y=38.8$;

$\cos 40^\circ=\dfrac{x}{NP}$, $x=46.3$

5. $RT^2=14^2+18^2-2(14)(18)\cos 34^\circ$, $RT=10.1$ cm;

$x^2=RT^2+16^2-2(RT)(16)\cos 65^\circ$, $x=14.9$

6. $\dfrac{51}{UW}=\cos 27^\circ$, $UW=57.24$ cm, $y^2=UW^2+46^2-2(UW)(46)\cos 51^\circ$, $y=45.6$

7. Draw 畫 ZC // YA, $\angle ZCB=45^\circ$, $\angle BZC=65^\circ$, $\dfrac{q}{\sin 45^\circ}=\dfrac{28-20}{\sin 65^\circ}$, $q=6.2$;

$\dfrac{ZC}{\sin 70^\circ}=\dfrac{8}{\sin 65^\circ}$, $r=ZC=8.3$

8. Draw 畫 EG // FC, $\angle DGE=58^\circ$, $\angle DEG=54^\circ$, $\dfrac{DG}{\sin 54^\circ}=\dfrac{7.2}{\sin 58^\circ}$, $DG=6.87$ cm;

$x=10.5-6.87=3.6$

9. $x^2=6^2+6^2-2(6)(6)\cos\angle ADC$, $x^2=72-72\cos\angle ADC$ … (1)

$x^2=3^2+8^2-2(3)(8)\cos(180^\circ-\angle ADC)$, $1.5x^2=109.5+72\cos\angle ADC$ … (2)

(1) + (2): $2.5x^2=181.5$, $x=8.5$

10. $\angle R=55^\circ$, $\dfrac{QR}{\sin 25^\circ}=\dfrac{8}{\sin 55^\circ}$, $QR=4.1274$ cm, $\therefore$ Area $=\dfrac{1}{2}(8)(4.1274)=16.3\text{ cm}^2$

11. $\dfrac{BD}{\sin 117^\circ}=\dfrac{15}{\sin 29^\circ}$, $BD=27.5677$ cm; $\dfrac{20+14+27.5677}{2}=30.78385$;

area of ΔCBD 面積 $=\sqrt{30.78385(3.21615)(10.78385)(16.78385)}$

$=133.86355\text{cm}^2$, $\angle DBA=34^\circ$, $\therefore$ area of ΔDBA面積 $=\dfrac{1}{2}(15)(27.5677)\sin 34^\circ$

$=115.6175\text{ cm}^2$, $\therefore$ The area required $=249.5\text{ cm}^2$

12. $DF=\sqrt{6^2+9^2-2(6)(9)\cos 92^\circ}=10.9895$ cm

area of ΔDCF面積 $=\sqrt{14.49475(3.49475)(7.49475)(3.50525)}=36.4797\text{ cm}^2$

$\therefore$ The required area $=\dfrac{1}{2}(6)(9)\sin 92^\circ+36.4797=63.5\text{ cm}^2$

3-7

1. $12^2=x^2+22^2-2(x)(22)\cos 30^\circ$, $x=23.8$ or 或 14.3
2. $35^2=x^2+26^2-2(x)(26)\cos 65^\circ$, $x=36.9$ or 或 -14.9 (rejected 捨去)
3. $27^2=x^2+32^2-2(x)(32)\cos 32^\circ$, $x=48.1$ or 或 6.1
4. $7^2=x^2+17^2-2(x)(17)\cos 26^\circ$, no solution 沒有解
5. $8^2=x^2+7^2-2(x)(7)\cos 42^\circ$, $x=11.7$ or 或 -1.3 (rejected 捨去)
6. $16^2=x^2+24^2-2(x)(24)\cos 18^\circ$, $x=37.0$ or 或 8.7

3-8

1. $\dfrac{BD}{\sin 20^\circ}=\dfrac{32}{\sin(50^\circ-20^\circ)}$, $BD=21.9$ cm; $x=BD\cos 50^\circ=14.1$

2. Let 設 $PS = y$ cm, $\angle SPQ = \theta$
   $y^2 = 20^2 + 25^2 - 2(20)(25)\cos 12°$, $y = 6.8$;
   $\frac{20}{\sin\theta} = \frac{y}{\sin 12°}$, $\theta = 37.4°$ or 或 $142.6°$ (rejected 捨去), $x = 25\cos\theta = 19.9$
3. $a = 20°$, $b = 180° - 160° = 20°$, $x^2 = 15^2 + 30^2 - 2(15)(30)\cos(20° + 20°)$, $x = 20.9$
4. $a = 45°$, $b = 360° - 328° = 32°$, $x^2 = 130^2 + 68^2 - 2(130)(68)\cos(45° + 32°)$, $x = 132.5$
5. $a = 32°$, $b = 180° - 120° = 60°$, $c = 60°$, $d = 360° - 255° - 60° = 45°$
   $\frac{x}{\sin(32° + 60°)} = \frac{275}{\sin 45°}$, $x = 388.7$
6. $\angle BAC = 16°$, $\angle BCA = 23°$, $\therefore \angle B = 141°$
   $\frac{170}{\sin 141°} = \frac{x}{\sin 23°} = \frac{y}{\sin 16°}$, $\therefore x = 105.5$, $y = 74.5$

3-9

1. $\angle CAD = 50° - 40° = 10°$, $\frac{AD}{\sin 40°} = \frac{50}{\sin 10°}$, $AD = 185.0833$ m
   $AB = AD\sin 50° = 142$ m
2. (a) $\angle BOC = 295° - 175° = 120°$, $\frac{8}{\sin\angle BCO} = \frac{30}{\sin 120°}$, $\angle BCO = 13.4°$
   (b) $13.4° + (180° - 175°) = 18.4°$ $\therefore$ N18.4°W
3. $\angle QPR = 104°$, $\angle R = 46°$, $\frac{PQ}{\sin 46°} = \frac{10}{\sin 104°}$, $PQ = 7.41$ m
   $\therefore$ Height of the tree 樹的高度 $= PQ\sin 30° = 3.71$ m
4. (a) $\angle BAC = 180° - 75° - 40° = 65°$
   (b) $BC^2 = 150^2 + 120^2 - 2(150)(120)\cos 65°$, $BC = 147.3$ m
5. (a) $\angle PRS = 180° - 90° - 35° = 55°$ (b) $\frac{PS}{\sin 55°} = \frac{100}{\sin 95°}$, $PS = 82.2$ m
   (c) $\angle PSQ = 60°$, $h = PS\sin 60° = 71.21$; $SQ = PS\cos 60° = 41.1$ m
6. (a) $\angle ADB = 30°$, $\angle ABD = 115°$, $\frac{AD}{\sin 115°} = \frac{5.2}{\sin 30°}$, $AD = 9.43$ km
   (b) $\frac{BD}{\sin 35°} = \frac{5.2}{\sin 30°}$, $BD = 5.97$ km
   (c) Shortest distance 最短距離
   = Perpendicular distance from D to AB (由 D 至 AB 的垂直距離)
   $= BD\sin 65° = 5.41$ km
7. (a) $\angle PQR = 180° - 90° - 25° = 65°$
   (b) $PQ = 2 \times 60 = 120$km, $PR^2 = 120^2 + 80^2 - 2(120)(80)\cos 65°$, $PR = 113$ km
   (c) $\frac{80}{\sin(25° + \theta)} = \frac{PR}{\sin 65°}$, $25° + \theta = 40.1°$, $\theta = 15.1°$
   $\therefore$ Compass bearing of R from P is S15.1°E. 由 P 至 R 的羅盤方位角是S15.1°E 。
8. (a) $\angle PQR = 105°$
   (b) $PR = 3 \times \frac{20}{60} = 1$ km

(c) $\angle RPQ = 40°$, $\dfrac{RQ}{\sin 40°} = \dfrac{1}{\sin 105°}$, $RQ = 0.6655$

$\therefore$ Time taken 所需時間 $= \dfrac{RQ}{3} = 0.2218$ hours $= 14$ minutes

3-10

1. (a) $\angle CBT = 90° - 25° = 65°$, $BC = \dfrac{60}{\cos 65°} = 141.9721$ km

Speed of the car 車速 $= \dfrac{BC}{1.5} = 94.7$ km/h

Shortest distance 最短距離 $= 60\sin 65° = 54.4$ km

(b) Time taken 所需時間 $= \dfrac{60\cos 65°}{94.65} = 0.2679$ hours $= 16$ minutes

$\therefore$ The car will be nearest to the tower at 1:16 p.m.
該車會於下午 1:16 最接近塔樓。

2. (a) $\angle BCP = 315° - 180° - 75° = 60°$, $\angle BPC = 75° - 35° = 40°$,
$\angle B = 180° - 60° - 40° = 80°$

$\dfrac{PB}{\sin 60°} = \dfrac{PC}{\sin 80°} = \dfrac{500}{\sin 40°}$, $PB = 674$ m, $PC = 766$ m

(b) $\dfrac{1}{2}(500)(QB)\sin 80° = 100000$, $QB = 406$ m

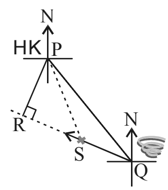
Distance he travels 他行走的距離 $= PB - QB = 267$m

(c) $QC^2 = QB^2 + 500^2 - 2(QB)(500)\cos 80°$, $QC = 586.89$ m

Cost 費用 $= QC \times 400 = \$235000$

3. (a) (i) $\angle PQR = 65° - 45° = 20°$,

Shortest distance 最短距離 $= PR = 650\sin 20° = 222$ miles



(ii) Time taken 所需時間 $= \dfrac{650\cos 20°}{15} = 41$ hours 小時

$\therefore$ The typhoon will be nearest to Hong Kong at 5:00a.m. on 3rd June.
颱風會於 6 月 3 日早上 5:00 最接近香港。

(b) $500^2 = SQ^2 + 650^2 - 2(SQ)(650)\cos 20°$

$SQ = 163$ or 或 1060 (rejected 捨去). Time taken 所需時間 $= \dfrac{SQ}{15} = 11$ hours 小時

$\therefore$ It will be hoisted at 11:00p.m. on 1st June. 會於 6 月 1 日晚上 11:00 懸掛。

4. (a) $\tan 38° = \dfrac{x}{h}$, $x = h\tan 38°$; $\tan 28° = \dfrac{y}{h}$, $y = h\tan 28°$

$\tan\angle CQP = \dfrac{h}{x+y} = \dfrac{h}{h(\tan 38° + \tan 28°)} = \dfrac{1}{\tan 38° + \tan 28°}$, $\angle CQP = 37.3°$

(b) $90° - \angle CQP = 52.7°$.

$\therefore$ The bearing of P from Q is N52.7°W. 由 Q 至 P 的方向是 N52.7°W。

3-11

1. Let 設 $s=\frac{6+7+11}{2}=12$, area 面積 $=\sqrt{12(12-6)(12-7)(12-11)}=6\sqrt{10}$
2. $\frac{\sin\angle BCA}{10}=\frac{\sin 40°}{7}$, $\sin\angle BCA=0.918$, $\angle BCA=66.7°$ or 或 $113.3°$
3. $x=\sqrt{6^2+9^2-2(6)(9)(-\frac{5}{6})}=3\sqrt{23}$
4. $\frac{1}{2}(7)(12)\sin\angle BAC=24$, $\sin\angle BAC=0.571$, $\angle BAC=34.8°$ or 或 $145.2°$
5. $8^2=a^2+(3a)^2-2a(3a)\cos 55°$, $64=a^2(1+9-6\cos 55°)$, $a=3.12$
6. $\cos\theta=\frac{1^2+1^2-a^2}{2(1)(1)}=\frac{2-a^2}{2}$
7. $\cos\angle BCA=\frac{7^2+6^2-9^2}{2(7)(6)}=\frac{1}{21}$, $\cos y=-\frac{1}{21}$
8. $22^2=x^2+16^2-2x(16)\cos 44°$, $x^2-23.02x-228=0$, $x=30.5$ or 或 -7.48 (rej.捨去)
9. $\angle ABC=180°-65°-38°=77°$, $\frac{AB}{\sin 38°}=\frac{8}{\sin 77°}$, $AB=5.05$,

   area 面積 $=\frac{1}{2}(5.05)(8)\sin 65°=18.3\text{ cm}^2$
10. $\cos\theta=\frac{11^2+20^2-23^2}{2(11)(20)}=-\frac{1}{55}$, $\sin\theta=\sqrt{1-(-\frac{1}{55})^2}=\frac{12\sqrt{21}}{55}$ $(\because 0°<\theta<180°,\ \sin\theta>0)$

3-12

1. $QR=9$, $\cos\angle PQR=\frac{12^2+9^2-18^2}{2(12)(9)}=-\frac{11}{24}$, $\angle PQR=117.3°$, $\angle QPS=180°-117.3°=62.7°$
2. Let E be a point on AD such that $BE // CD$. 設 E 為 AD 上的一點使得 $BE // CD$。

   $BE=6\text{ cm}$, $AE=8-5=3\text{ cm}$, $\angle BEA=54°$; $y^2=3^2+6^2-2(3)(6)\cos 54°$, $y=4.88$
3. Let T be a point on PS such that $RT // QP$. 設 T 為 PS 上的一點使得 $RT // QP$。

   $\angle QPT=180°-118°=62°$, $\therefore \angle RTS=62°$; $\angle RST=180°-135°=45°$; $\angle TRS=73°$;

   $ST=16-10=6\text{ cm}$, $\frac{x}{\sin 62°}=\frac{6}{\sin 73°}$, $x=5.54$
4. $\angle ABD=37°$, $\angle BDC=74°$, $\angle BCD=38°$, $\frac{BC}{\sin 74°}=\frac{5}{\sin 38°}$, $BC=7.81$
5. (a) $\because AE:EC=1:1$, $\therefore$ Area of ΔAED 面積: Area of ΔDEC 面積 $=1:1$ (height eq.等高)

   (b) Area of ΔAED 面積 $=\frac{1}{2}(7)(4)\sin 130°=10.72\text{ cm}^2$,

   $\therefore$ Area of the parallelogram $ABCD$ (平行四邊形 $ABCD$ 的面積) $=4\times 10.72=42.9\text{ cm}^2$
6. Let the height from B to AC be h. 設由 B 至 AC 的高為 h。

   $\frac{h}{12}=\sin 45°=\frac{\sqrt{2}}{2}$, $h=6\sqrt{2}$. $a>6\sqrt{2}$ but 但 $a<12$, $\therefore 6\sqrt{2}<a<12$

4-1

1. *DB, DC*
2. (a) *PSFC, QDER* (b) *QR, PS, CF, DE*

(c) Yes. $\because FS \perp PS$, $FS \perp SR$ $\therefore FS$ is perpendicular to $PSRQ$
$\because$ SQ is on plane $PSRQ$ (SQ *位於平面* $PSRQ$ 上), $\therefore \angle FSQ = 90°$

3. Yes.是 $\because AB \perp AD$ & $AB \perp AC$, $\therefore BA \perp$ plane平面 $ACDE$, $\therefore \angle BAE = 90°$

4. (a) May not be. ∠DEA = 90°, but the value of ∠DEB is not known.
不一定，∠DEA = 90°，但不知道∠DEB 的值。
(b) ∠AEB, ∠AED, ∠AEC

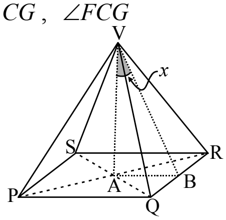
4-2

1. (i) BC, $\angle FBC$ (ii) DE, $\angle AED$ (iii) BD, $\angle EBD$
(iv) EC, $\angle BEC$

2. (i) AE, $\angle FAE$ (ii) EF, $\angle AFE$ (iii) BF, $\angle AFB$
(iv) BC, $\angle FCB$ (v) CG, $\angle FCG$ (vi) FG, $\angle CFG$

3. (a) (i) $\angle VQA$
(ii) $\angle VBA$

(b)



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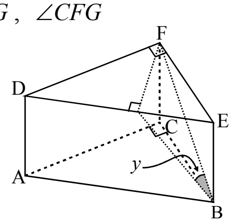
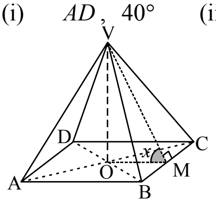


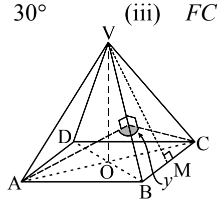
4-3

1. (a) (i) AB, 90° (ii) AB, 45° (iii) HG, 45°
(b) (i) AD, 40° (ii) BE, 30° (iii) FC, 110° (iv) EF, 90°

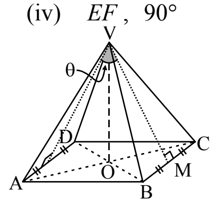
2. (a)



(b)



(c)



4-4

1. (a) $AH = \sqrt{4^2 + 7^2} = \sqrt{65}$, $9^2 + (\sqrt{65})^2 = BH^2$, $BH = \sqrt{146}$

(b) $\tan \angle EAH = \dfrac{EH}{AE}$, $\angle EAH = 60.3°$; $\cos \angle HBF = \dfrac{BF}{HB}$, $\angle HBF = 70.7°$;
$\cos \angle ABH = \dfrac{AB}{HB}$, $\angle ABH = 41.9°$

(c) (i) HE ⊥ ABFE, ∴ the angle required 所求的角 = ∠HAE.
$\tan \angle HAE = \dfrac{7}{4}$, $\angle HAE = 60.3°$

(ii) HE ⊥ ABFE, ∴ the angle required 所求的角 = ∠HBE.
$BE = \sqrt{9^2 + 4^2} = \sqrt{97}$; $\tan \angle HBE = \dfrac{7}{\sqrt{97}}$, $\angle HBE = 35.4°$

2. (a) $BG = \sqrt{20^2 + 15^2} = 25$ cm, $DB = \sqrt{48^2 + 20^2} = 52$ cm, $GD = \sqrt{48^2 + 15^2} = 3\sqrt{281}$ cm

(b) (i) $\tan \angle CGD = \dfrac{CD}{CG}$, $\angle CGD = 72.6°$; (ii) $\tan \angle GDC = \dfrac{GC}{CD}$, $\angle GDC = 17.4°$

3. (a) Rectangle 長方形

(b) $5^2+6^2=CH^2$, $CH=\sqrt{61}$; $(\sqrt{61})^2+8^2=EC^2$, $EC=5\sqrt{5}$; $EK=\frac{1}{2}EC=\frac{5\sqrt{5}}{2}$

(c) $8^2=(\frac{5\sqrt{5}}{2})^2+(\frac{5\sqrt{5}}{2})^2-2(\frac{5\sqrt{5}}{2})(\frac{5\sqrt{5}}{2})\cos\theta$, $\theta=91.4°$

4. (a) $x^2+(2x)^2=PM^2$, $PM=\sqrt{5}x$; $(2x)^2+(\sqrt{5}x)^2=AM^2$, $AM=3x$;

$\sin\angle AMP=\frac{AP}{AM}=\frac{2x}{3x}=\frac{2}{3}$, $\angle AMP=41.8°$

(b) Isosceles triangle 等腰三角形

(c) $\cos\theta=\frac{AM^2+BM^2-AB^2}{2(AM)(BM)}=\frac{(3x)^2+(3x)^2-(2x)^2}{2(3x)(3x)}=\frac{14x^2}{18x^2}=\frac{14}{18}=\frac{7}{9}$

4-5

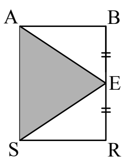
1. (a) (i) $\angle BRS=90°$

(ii) Let the side of the cube be x. 設正方體的邊長為 x。

$BQ^2+QR^2=BR^2$, $BR=\sqrt{2}x$, $ER=\frac{\sqrt{2}}{2}x$; $SR^2+ER^2=SE^2$, $SE=\sqrt{\frac{3}{2}}x$

$\sin\angle RSE=\frac{ER}{SE}=\frac{\sqrt{2}}{2}x\div\sqrt{\frac{3}{2}}x=\frac{\sqrt{3}}{3}$

(b)

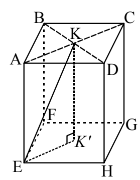


$\cos\angle AES=\frac{AE^2+SE^2-AS^2}{2(AE)(SE)}=\frac{(\sqrt{\frac{3}{2}}x)^2+(\sqrt{\frac{3}{2}}x)^2-(\sqrt{2}x)^2}{2(\sqrt{\frac{3}{2}}x)(\sqrt{\frac{3}{2}}x)}=\frac{1}{3}$

2. (a) (i) $(2p)^2+(2q)^2=AC^2$, $AC=2\sqrt{p^2+q^2}$; $AK=\frac{1}{2}AC=\sqrt{p^2+q^2}$;

$AK^2+AE^2=EK^2$, $(\sqrt{p^2+q^2})^2+(2r)^2=EK^2$, $EK=\sqrt{p^2+q^2+4r^2}$

(ii) $\cos\angle KEA=\frac{AE}{KE}=\frac{2r}{\sqrt{p^2+q^2+4r^2}}$



(b) Let the projection of K on $EFGH$ be K'.
設 K 於平面 $EFGH$ 的投影為 K'。
$\angle KEK'=60°$, $KK'=2r$,

$\sin 60°=\frac{KK'}{EK}$, $\frac{\sqrt{3}}{2}=\frac{2r}{\sqrt{p^2+q^2+4r^2}}$, $3p^2+3q^2-4r^2=0$

3. (a) $\tan 30°=\frac{AD}{DH}$, $DH=\sqrt{3}AD$; $\tan 60°=\frac{DC}{AD}$, $DC=\sqrt{3}AD$; $\tan\theta=\frac{\sqrt{3}AD}{\sqrt{3}AD}=1$

(b) $\tan 60°=\frac{BF}{EF}$, $BF=\sqrt{3}EF$; $\tan 45°=\frac{FG}{EF}$, $FG=EF\tan 45°=EF$

$EF^2+(\sqrt{3}EF)^2=GB^2$, $GB=2EF$; $\cos\theta=\frac{FG}{GB}=\frac{EF}{2EF}=\frac{1}{2}$

4-6

1. (a) $CD=24\cos 60°=12$ cm; $CF=CD\sin 30°=6$ cm

(b) $BE = CF = 6$ cm, $\sin\angle BDE = \frac{BE}{BD} = \frac{6}{24}$, $\angle BDE = 14.5°$

2. (a) $MNRS, MNQP, PQRS$

(b) $5^2 + (2\sqrt{61})^2 = NP^2$, $NP = \sqrt{269}$; $12^2 + 10^2 = MP^2$, $MP = 2\sqrt{61}$

(c) (i) $\tan\angle PMS = \frac{PS}{MS} = \frac{10}{12}$, $\angle PMS = 39.8°$

(ii) $\sin\angle NPR = \frac{NR}{NP} = \frac{12}{\sqrt{269}}$, $\angle NPR = 47.0°$

(iii) $\sin\angle SNP = \frac{SP}{NP} = \frac{10}{\sqrt{269}}$, $\angle SNP = 37.6°$

3. (a) $DE = \sqrt{12^2 + 9^2} = 15$ cm; $AF = \sqrt{6^2 + 12^2} = 6\sqrt{5}$ cm; $BF = \sqrt{9^2 + 6^2} = 3\sqrt{13}$ cm

(b) (i) $\cos\angle AFB = \frac{(6\sqrt{5})^2 + (3\sqrt{13})^2 - 15^2}{2(6\sqrt{5})(3\sqrt{13})}$, $\angle AFB = 75.6°$

(ii) $\tan\angle AFC = \frac{AC}{FC} = \frac{12}{6}$, $\angle AFC = 63.4°$

(iii) $\tan\angle BFE = \frac{BE}{FE} = \frac{6}{9}$, $\angle BFE = 33.7°$

(iv) $\tan\angle BFC = \frac{BC}{FC} = \frac{9}{6}$, $\angle BFC = 56.3°$

4-7

1. (a) $BC = 4$cm, $AB^2 + BC^2 = AC^2$, $12^2 + 4^2 = AC^2$, $AC = 4\sqrt{10}$cm

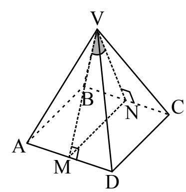
$AC^2 + CR^2 = AR^2$, $(4\sqrt{10})^2 + 4^2 = AR^2$, $AR = 4\sqrt{11}$cm

(b) (i) $\sin\angle ARB = \frac{AB}{AR} = \frac{12}{4\sqrt{11}}$, $\angle ARB = 64.8°$

(ii) $\tan\angle ACB = \frac{AB}{BC} = \frac{12}{4}$, $\angle ACB = 71.6°$ (iii) $\angle ACB = 71.6°$

2. Draw 畫 $VM \perp AD, VN \perp BC$.

The angle required 所求的角 $= \angle MVN$.



$VM = VN = \sqrt{15^2 - 5^2} = \sqrt{200}$; $\cos\theta = \frac{(\sqrt{200})^2 + (\sqrt{200})^2 - 10^2}{2(\sqrt{200})(\sqrt{200})}$,

$\theta = 41.4°$

$\therefore$ Angle between VAD and VBC (VAD 與 VBC 的夾角) = 41.4°

3. (a) $AB^2 + BC^2 = AC^2$, $20^2 + 20^2 = AC^2$, $AC = 20\sqrt{2}$

(b) Let the mid point of BC be M. 設 M 為 BC 的中點。

$PM^2 + MC^2 = AC^2$, $PM^2 + 10^2 = 26^2$, $PM = 24$ cm

area of ΔPBC面積 $= \frac{24 \times 20}{2} = 240$ cm^2

$\frac{CE \times PB}{2} = 240$, $\frac{CE \times 26}{2} = 240$, $CE = \frac{240}{13}$cm

(c) $\cos\angle AEC = \frac{AE^2 + CE^2 - AC^2}{2(AE)(CE)} = \frac{(\frac{240}{13})^2 + (\frac{240}{13})^2 - (20\sqrt{2})^2}{2(\frac{240}{13})(\frac{240}{13})}$, $\angle AEC = 100°$

4. (a) $\cos\angle VQR = \dfrac{9\div 2}{18} = \dfrac{1}{4}$, $\angle VQR = 75.5°$;

$HR = RQ\sin\angle VQR = 8.71$ cm; $HQ = RQ\cos\angle VQR = 2.25$ cm

(b) $\cos\angle VQP = \dfrac{14\div 2}{18} = \dfrac{7}{18}$, $\angle VQP = 67.1°$;

$HK = HQ\tan\angle VQP = 5.33$ cm; $\dfrac{HQ}{KQ} = \cos\angle VQP$, $KQ = 2.25\div\dfrac{7}{18} = 5.79$ cm

(c) $KR = \sqrt{KQ^2 + RQ^2} = 10.7$ cm

The required angle 所求的角 $= \angle RHK$,

$\cos\angle RHK = \dfrac{HK^2 + HR^2 - KR^2}{2(HK)(HR)} = \dfrac{5.33^2 + 8.71^2 - 10.7^2}{2(5.33)(8.71)} = -0.1107$, $\angle RHK = 96°$

4-8

1. $AC = \dfrac{20}{\tan 37°} = 26.5\text{cm}$, $AB = \dfrac{20}{\tan 37°} = 49.5$ cm,

$BC^2 = AC^2 + AB^2 - 2(AC)(AB)\cos 50°$, $BC = 38.3$ cm

2. (a) $\tan 45° = \dfrac{x}{BC}$, $BC = \dfrac{x}{\tan 45°} = x$, $\tan 30° = \dfrac{x}{BD}$, $BD = \dfrac{x}{\tan 30°} = \sqrt{3}x$,

$CD^2 = BC^2 + BD^2 - 2(BC)(BD)\cos 60°$, $48^2 = x^2 + (\sqrt{3}x)^2 - 2(x)(\sqrt{3}x)\cos 60°$,

$x = 31.9$ m

(b) $BD = \sqrt{3}x = 55.2\text{m}$, $\cos\theta = \dfrac{BC^2 + CD^2 - BD^2}{2(BC)(CD)} = \dfrac{31.9^2 + 48^2 - 55.2^2}{2(31.9)(48)}$, $\theta = 84.9°$

3. (a) $\tan 38° = \dfrac{VA}{AB} = \dfrac{h}{AB}$, $AB = \dfrac{h}{\tan 38°}$, $\tan 70° = \dfrac{VA}{AC} = \dfrac{h}{AC}$, $AC = \dfrac{h}{\tan 70°}$

$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos 42°$

$24^2 = (\dfrac{h}{\tan 38°})^2 + (\dfrac{h}{\tan 70°})^2 - 2(\dfrac{h}{\tan 38°})(\dfrac{h}{\tan 70°})\cos 42°$

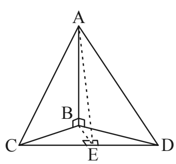
$24^2 = h^2(\dfrac{1}{\tan^2 38°} + \dfrac{1}{\tan^2 70°} - 2\cdot\dfrac{1}{\tan 38°}\cdot\dfrac{1}{\tan 70°}\cdot\cos 42°)$, $h = 23.1$

(b) $AB = \dfrac{23.1}{\tan 38°} = 29.6\text{cm}$, $AC = \dfrac{23.1}{\tan 70°} = 8.41\text{cm}$,

$\cos\angle ACB = \dfrac{AC^2 + BC^2 - AB^2}{2(AC)(BC)} = \dfrac{8.41^2 + 24^2 - 29.6^2}{2(8.41)(24)}$, $\angle ACB = 124°$

(c) area of ΔABC 面積 $= \dfrac{1}{2}\cdot AB\cdot AC\cdot\sin\angle BAC = \dfrac{1}{2}(29.6)(8.41)\sin 42° = 83.3\text{ cm}^2$

4.

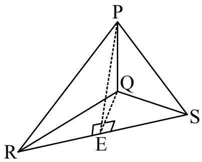


Let E be a point on CD such that AE is a line of greatest slope.
設 E 為 CD 上的一點，且 AE 為最大傾斜線。

$\cos\angle BCD = \dfrac{5^2 + 10^2 - 7^2}{2(5)(10)}$, $\angle BCD = 40.5°$

$\therefore$ $BE = 5\sin 40.5° = 3.25$ cm

The required angle 所求的角 $= \angle AEB$, $\tan\angle AEB = \dfrac{AB}{BE} = \dfrac{3.25}{8}$, $\angle AEB = 67.9°$

5. 

Let E be a point on RS such that PE is a line of greatest slope.
設 E 為 RS 上的一點，且 PE 為最大傾斜線。
$\therefore\ PE \perp RS,\ QE \perp RS.$
$RS^2 = 48^2 + 30^2 - 2(48)(30)\cos 120°$, $RS = \sqrt{4644}$;
$\therefore\ \cos\angle QRS = \dfrac{48^2 + (\sqrt{4644})^2 - 30^2}{2(48)(\sqrt{4644})}$, $\angle QRS = 22.4°$
$QE = RQ \sin 22.4° = 18.3$
The required angle 所求的角 $= \angle PEQ$, $\tan\angle PEQ = \dfrac{PQ}{QE} = 1.4754$, $\angle PEQ = 55.9°$

4-9

1. (a) $VQ^2 + QR^2 = VR^2, VR = 10$ cm;
$PR^2 = VP^2 + VR^2 - 2\cdot VP\cdot VR\cos\angle PVR$, $PR = 2\sqrt{109}$
(b) $PR^2 = PQ^2 + RQ^2 - 2\cdot PQ\cdot RQ\cos\angle PQR$, $\angle PQR = 90°$
$PV^2 = PQ^2 + VQ^2 - 2\cdot PQ\cdot VQ\cos\angle VQP$, $\angle VQP = 33.1°$
(c) Yes, it is because their intersection line is QR and $VQ \perp QR$ and $PQ \perp QR$.
是，因為它們相交於 QR，並且 $VQ \perp QR$ 及 $PQ \perp QR$。
2. (a) $AB^2 + BC^2 = AC^2$, $BC = 8$ cm; $AC^2 + CD^2 = AD^2$, $AD = 26$ cm;
$BD^2 = AB^2 + AD^2 - 2\cdot AB\cdot AD\cos\angle BAD$, $BD = 29.5$ cm
(b) $BD^2 = BC^2 + DC^2 - 2\cdot BC\cdot DC\cos\angle BCD$, $\angle BCD = 126°$
$\because \angle BCD \neq 90°$, therefore, BC is not perpendicular to CD
所以 BC 不垂直於 CD。
$\therefore\ \angle ACB$ is not the angle between planes BCD and ACD
$\angle ACB$ 不是平面 BCD 和 ACD 之間的角。
(c) (i) $BC^2 = CD^2 + BD^2 - 2\cdot CD\cdot BD\cos\angle CDB$, $\angle CDB = 12.6°$
$\tan\angle CDS = \dfrac{SC}{CD}$, $SC = 5.37$ cm; $SC^2 + CD^2 = SD^2$, $SD = 24.6$ cm
(ii) $\dfrac{\sin\angle ADB}{AB} = \dfrac{\sin\angle BAD}{BD}$, $\angle ADB = 10.2°$;
$AS^2 = SD^2 + AD^2 - 2\cdot SD\cdot AD\cos\angle ADS$, $AS = 4.69$ cm
(iii) $AS^2 = SC^2 + AC^2 - 2\cdot SC\cdot AC\cos\angle ACS$, $\angle ACS = 6.16°$
3. (a) $\dfrac{\sin\angle BCD}{BD} = \dfrac{\sin\angle BDC}{BC}$, $BD = 29.6$; $\dfrac{\sin\angle DBC}{CD} = \dfrac{\sin\angle BDC}{BC}$, $CD = 31.6$
$AC = \sqrt{28^2 + 21^2} = 35$, $AB^2 + BD^2 = AD^2$, $AD = 40.8$
(b) $\angle DBC = 180° - 40° - 65° = 75°$,
area of ΔBCD 的面積 $= \dfrac{1}{2}(29.6)(21)\sin 75° = 300\text{ cm}^2$
(c) (i) $\because AB \perp BD$ and $AB \perp BC$, $\therefore\ AB \perp$ plane 平面BCD.
(ii) volume 體積 $= \dfrac{1}{3}(300)(28) = 2800\text{ cm}^3$
(d) (i) $s = \dfrac{AD + AC + CD}{2} = \dfrac{40.8 + 35 + 31.6}{2} = 53.7$,
$\therefore$ Area of ΔACD (ΔACD 的面積) $= \sqrt{53.7(12.9)(18.7)(22.1)} = 535\text{ cm}^2$

(ii) volume of tetrahedron 四面體的體積 $= \frac{1}{3}(535)(BE) = 2800$, $\therefore\ BE = 15.7$ cm

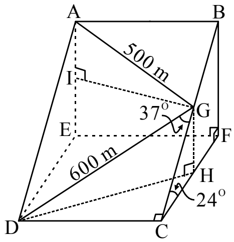
(iii) The required angle 所求的角 $= \angle BAE$; $\sin\angle BAE = \frac{BE}{AB}$, $\angle BAE = 34.1°$

4-10

1. (a) $AD = AR = 90$ cm, $DR^2 = 90^2 + 90^2 - 2\cdot 90\cdot 90\cos 36°$, $DR = 55.6$ cm;
$CS = DR = 55.6$ cm, $AC = AS = \sqrt{90^2 + 210^2} = 228.47$,
$\cos\angle CAS = \frac{AC^2 + AS^2 - CS^2}{2(AC)(AS)}$, $\angle CAS = 14.0°$

(b) (i) $SE = 90\sin 36° = 52.9$ cm (ii) $\sin\angle EAS = \frac{SE}{AS}$, $\angle EAS = 13.4°$

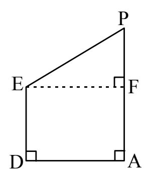
2. (a) AB = DC, $\therefore$ $500\sin\angle AGB = 600\sin 37°$, $\angle AGB = 46.2°$



(b) Let H be a point on CF such that $GH \perp CF$.
設 H 在 CF 上使得 $GH \perp CF$ 。
$CG = 600\cos 37° = 479.18$ m;
vertical height of G (G 的高度) $= HG = CG\sin 24°$
= 194.9 m
$\sin\angle GDH = \frac{HG}{600}$, $\angle GDH = 19.0°$

(c) $\angle AGB = 46.2°$, $BG = 500\cos 46.2° = 346.07$ m;
vertical height of A (A 的高度)= $AE = BF$
= $(BG + CG)\sin 24° = 335.66$ m
Let I be a point on AE such $EI = HG$.
設 I 在 AE 上使得 $EI = HG$。
$AI = BF - HG =$ 335.66 − 194.9 =140.76m; $\sin\angle AGI = \frac{AI}{500}$, $\angle AGI = 16°$

3. (a) $\angle CAB = 97°$, $\frac{\sin 53°}{AC} = \frac{\sin 30°}{AB} = \frac{\sin 97°}{900}$, $AC = 724.2$ m,
$AB = 453.4$ m, $\angle PCA = 27°$, $PA = AC\tan 27° = 369.0$ m



(b) $\tan\angle PBA = \frac{PA}{AB}$, $\angle PBA = 39.1°$

(c) (i) shortest distance from A
與 A 點的最短距離 $= AD = AC\sin 30° = 362.1$ m

(ii) Let E and F be the points 1.5 m above D and A respectively.
設 E 和 F 點分別在 D 和 A 點的 1.5 m 上方。
The required angle 所求的角 $= \angle PEF$.
$PF = PA - 1.5 = 367.5$ m, $\tan\angle PEF = \frac{PF}{EF}$, $\angle PEF = 45.4°$

4-11

1. (a) $\cos\angle ABC = \frac{20^2 + 28^2 - 15^2}{2(20)(28)}$, $\angle ABC = 31.1°$; $BD = 20\cos\angle ABC = 17.1$ cm;
$DC = 28 - BD = 10.9$ cm

(b) $BC^2 = 20^2 + 15^2 - 2 \cdot 20 \cdot 15 \cos 30°$, $BC = 10.3$ cm

(c) AD is the line of intersection of BDA and CDA,
(AD 是 BDA 和 CDA 的相交線) $BD \perp AD$ and $CD \perp AD$.
$\therefore$ $\angle BDC$ is the angle between BDA and CDA. ($\angle BDC$ 是 BDA 和 CDA 間的角)

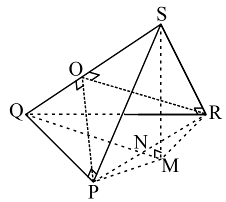
$\cos \angle BDC = \dfrac{BD^2 + DC^2 - BC^2}{2(BD)(DC)}$, $\angle BDC = 34.7°$

2. (a) $QS = \sqrt{10^2 + 10^2} = 14.1$ cm, $QM = \sqrt{QS^2 - 8^2} = 11.7$ cm

(b) $\because$ SM is perpendicular to the plane $PQRM$. (SM 垂直於平面 $PQRM$)
$\therefore$ $SM \perp PM$, $\angle SMP = 90°$; $PM = \sqrt{10^2 - 8^2} = 6$ cm

(c) $\cos \angle PQM = \dfrac{10^2 + QM^2 - 6^2}{2(10)(QM)}$, $\angle PQM = 31.0°$;
$\because$ symmetry 對稱, $\therefore$ $PN = NR$, $QM \perp PR$;
$PN = 10 \sin \angle PQM = 5.14$ cm,
$PR = 2 \times PN = 10.3$ cm

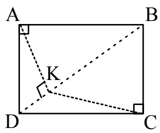
(d)



(e) The required angle 所求的角 $= \angle POR$

$PO = OR = \dfrac{1}{2}QS = 7.07$ cm, $\cos \angle POR = \dfrac{PO^2 + OR^2 - PR^2}{2(PO)(OR)}$, $\angle POR = 93.4°$

3. (a)



(b) (i) $BD = 10$ cm, $\angle ADB = 53.1°$

(ii) $AK = 6 \sin \angle ADB = 4.8$ cm;
$DK = 6 \cos \angle ADB = 3.6$ cm;
$KB = 10 - 3.6 = 6.4$ cm

(iii) $\angle BDC = 36.9°$;
$KC^2 = 8^2 + 3.6^2 - 2(8)(3.6)\cos 36.9°$,
$KC = 5.56$ cm

(c) (i) $AC = 7.34$ cm (ii) $\cos \angle BAC = \dfrac{8^2 + AC^2 - 6^2}{2(8)(AC)}$, $\angle BAC = 45.8°$

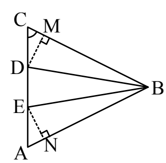
(iii) $a = 6,\ b = 8,\ c = 7.34,\ s = 10.67$; Area 面積 $= 21.1\ \text{cm}^2$

4. (a) (i) $CD = 4$ cm, $BD^2 = 4^2 + 12^2 - 2(4)(12)\cos 60°$, $BD = 10.6$ cm

$\cos \angle CDB = \dfrac{4^2 + 10.6^2 - 12^2}{2(4)(10.6)}$, $\angle CDB = 100.9°$

(ii) $\angle EDB = 180° - 100.9° = 79.1°$, $\angle DBE = 180° - 2 \times 79.1° = 21.8°$

(b) $DM = CD \sin 60° = 3.46$ cm,
$\because$ symmetry 對稱地, $\therefore$ $EN = DM = 3.46$ cm



(c) F: mid-pt of DE (DE 的中點),
G: projection of F on the horizontal ground
(G 為 F 於水平面的的投影).
$BF = BC \sin 60° = 10.4$ cm, $FG = DM = EN = 3.46$ cm,

$\sin \angle FBG = \dfrac{FG}{BF}$, $\angle FBG = 19.5°$

$\therefore$ The required angle 所求的角 $= 19.5°$

4-12

1. (a) $PS = 63\cos 22° = 58.4$ cm; $RS = 63\sin 22° = 23.6$ cm; In ΔRQS, $QS = 21.6$ cm

$\cos\angle SPQ = \dfrac{46^2 + PS^2 - QS^2}{2(46)(PS)}$, $\angle SPQ = 19.7°$;

$\cos\angle PQS = \dfrac{46^2 + QS^2 - PS^2}{2(46)(QS)}$, $\angle PQS = 114.6°$

(b) $90° - 19.7° = 70.3°$, bearing of R from P (由 P 至 R 的羅盤方位角): N70.3°E
$114.6° - 90° = 24.6°$, bearing of R from Q (由 Q 至 R 的羅盤方位角): N24.6°E

(c) Area of the shadow of PQR 影子的面積= Area of ΔPQS 面積

$= \dfrac{1}{2}(PQ)(PS)\sin\angle SPQ = 452\ \text{cm}^2$

2. (a) $AB = \dfrac{h}{\tan 60°} = \dfrac{h}{\sqrt{3}}$, $BD = \dfrac{h}{\tan 30°} = \sqrt{3}h$

In ΔABD (在 ΔABD 中), $600^2 = (\dfrac{h}{\sqrt{3}})^2 + (\sqrt{3}h)^2 - 2(\dfrac{h}{\sqrt{3}})(\sqrt{3}h)\cos(18° + 62°)$,

$h = 347$

(b) $BD = \sqrt{3}(347) = 601$ m, $\dfrac{BD}{\sin\angle BAD} = \dfrac{600}{\sin 80°}$, $\angle BAD = 80.8°$

(c) (i) $180° - 18° - \angle BAD = 81.2°$,
bearing of D from A (由 A 至 D 的羅盤方位角): S81.2°E

(ii) bearing of A from D (由 D 至 A 的羅盤方位角): N81.2°W

3. (a) $PK = 28\sin 42° = 18.7$ cm, Area of ΔPQR 面積= $\dfrac{1}{2}(28)(38)\sin 42° = 356\ \text{cm}^2$

(b) $LK = \dfrac{PK}{\tan 30°} = 32.5$ cm,

perpendicular distance from L to QR (由 L 至 QR 的垂直距離)= $LK\sin 70° = 30.5$ cm

Area of shadow LQR 影子面積 $= \dfrac{1}{2}(30.5)(38) = 576\ \text{cm}^2$

4-13

1. (a) No 不是 (b) Yes 是, $\because HG \perp CBGF$
(c) No 不是 (d) Yes 是, $\because HA \perp ABCD$

2. $BC \perp CDEF$, $\therefore$ the required angle is (所求的角是) $\angle BEC$. $EF = 15$ cm,

$EC = \sqrt{15^2 + 10^2} = \sqrt{325}$ cm; $BC = \dfrac{10}{\tan 30°} = 10\sqrt{3}$ cm, $\tan\angle BEC = \dfrac{10\sqrt{3}}{\sqrt{325}}$, $\angle BEC = 44°$

3. $AB = y\cos\alpha$, $AE = y\sin\alpha$, $BF = y\sin\alpha$, $BC = y\sin\alpha\cos\theta$, $FC = y\sin\alpha\sin\theta$;

volume 體積 $= \dfrac{(y\sin\alpha\cos\theta)(y\sin\alpha\sin\theta)}{2}(y\cos\alpha) = \dfrac{1}{2}y^3\sin^2\alpha\cos\alpha\sin\theta\cos\theta$

4. Let 設 $DB = x$, $AB = \dfrac{x}{\tan 40°}$, $BC = \dfrac{x}{\tan 50°}$; $\tan\theta = \dfrac{AB}{BC} = 1.42$, $\theta = 54.9°$

5. $AC = \dfrac{x}{\tan 45°} = x$, $AB = \dfrac{x}{\tan 30°} = \sqrt{3}x$;

$50^2 = x^2 + (\sqrt{3}x)^2 - 2x(\sqrt{3}x)\cos 120°$, $2500 = x^2(1 + 3 + \sqrt{3})$, $x = 20.9$

6. Let P be the mid-point of AB. 設 P 為 AB 的中點。

$MB = 3$ cm, $BP = 2$ cm, $\therefore MP = \sqrt{3^2 + 2^2} = \sqrt{13}$ cm, $MN = \sqrt{8^2 + (\sqrt{13})^2} = \sqrt{77}$ cm

7. (a) $DG = 12$, $\tan\angle DHG = \dfrac{12}{17}$, $\angle DHG = 35.2°$

(b) $HF = \sqrt{17^2 + 10^2} = \sqrt{389}$, $\tan\angle CHF = \dfrac{12}{\sqrt{389}}$, $\angle CHF = 31.3°$

(c) $BD = \sqrt{389}$, $MD = \dfrac{\sqrt{389}}{2}$, $\tan\angle MGE = \dfrac{12}{(\frac{\sqrt{389}}{2})}$, $\angle MGE = 50.6°$

8. Let 設 $DE = x$, $EC = \dfrac{x}{\cos 45°} = \sqrt{2}x$, $AE = \dfrac{x}{\cos 60°} = 2x$, $DC = x\tan 45° = x$,

$AD = x\tan 60° = \sqrt{3}x$; $AC = \sqrt{x^2 + (\sqrt{3}x)^2} = 2x$;

$\cos\angle AEC = \dfrac{(2x)^2 + (\sqrt{2}x)^2 - (2x)^2}{2(2x)(\sqrt{2}x)}$, $\angle AEC = 69.3°$

9. Let M be a point on AC such that 設 M 為 AC 上的一點使得
$DM \perp AC$, $BM \perp AC$, then 則 $AM = CM = 2$ cm.

$DM = BM = \sqrt{4^2 - 2^2} = 2\sqrt{3}$, $\cos\angle DMB = \dfrac{(2\sqrt{3})^2 + (2\sqrt{3})^2 - 4^2}{2(2\sqrt{3})(2\sqrt{3})}$, $\angle DMB = 71°$

10. Let 設 $AC = x$, $CD = 2x$, $AD = \sqrt{x^2 + (2x)^2} = \sqrt{5}x$, $BD = \sqrt{5}x$,

$AB = \sqrt{(\sqrt{5}x)^2 + (\sqrt{5}x)^2} = \sqrt{10}x$; $\sin\angle ABC = \dfrac{x}{\sqrt{10}x}$, $\angle ABC = 18.4°$

11. (a) Let 設 $PQ = x$, T be the foot of perpendicular from V to the plane PQRS.
T 為由 V 至平面 PQRS 的垂足。

$PR = \sqrt{x^2 + x^2} = \sqrt{2}x$, $PT = \dfrac{\sqrt{2}x}{2}$, $VQ = VP = \dfrac{\sqrt{2}x}{2\cos 45°} = x$;

$\because VQ = VP = PQ = x$, $\therefore \angle PVQ = 60°$

(b) Let M and N be the mid-points of PS and QR respectively.
設 M 及 N 分別為 PS 及 QR 的中點。

$VN = VM = \sqrt{x^2 - (\frac{x}{2})^2} = \dfrac{\sqrt{3}x}{2}$; $\cos\angle MVN = \dfrac{(\frac{\sqrt{3}x}{2})^2 + (\frac{\sqrt{3}x}{2})^2 - x^2}{2(\frac{\sqrt{3}x}{2})(\frac{\sqrt{3}x}{2})}$, $\angle MVN = 70.5°$

12. Area of ΔABC 面積 $= \dfrac{8 \times 15}{2} = 60$ cm^2; $BC = \sqrt{8^2 + 15^2} = 17$ cm;

$\dfrac{AE \times 17}{2} = 60$, $AE = \dfrac{120}{17}$ cm; $\tan\theta = \dfrac{10}{\frac{120}{17}} = \dfrac{17}{12}$

13. $\because CM \perp AM$, $BM \perp AM$, $\therefore \angle CMB$ represents the required angle. $\angle CMB$是該交角。
Let 設 $MC = x$, then 則 $AC = AB = 2x$, $BC^2 = (2x)^2 + (2x)^2 - 2(2x)(2x)\cos 45°$,

$BC = 1.53x$; $\cos\angle CMB = \dfrac{x^2 + x^2 - (1.53x)^2}{2(x)(x)}$, $\angle CMB = 99.9°$

14. (a) $DC = \sqrt{60^2 + 25^2} = 65$, $AC^2 = 15^2 + 65^2 - 2(15)(65)\cos 120°$, $AC = 73.7$

(b) $AB = \sqrt{25^2 - 15^2} = 20$, $\cos\angle ABC = \dfrac{20^2 + 60^2 - (73.7)^2}{2(20)(60)}$, $\angle ABC = 126.4° \neq 90°$

$\therefore \angle ABD$ does not represent the angle. $\angle ABD$ 不是該交角。

15. (a) $QS = \dfrac{h}{\tan 60°} = \dfrac{\sqrt{3}h}{3}$ m, $RS = \dfrac{h}{\tan 30°} = \sqrt{3}h$ m;

$(\sqrt{3}h)^2 = (\frac{\sqrt{3}h}{3})^2 + 100^2 - 2(\frac{\sqrt{3}h}{3})(100)\cos 45°$,

$\frac{8}{3}h^2 + \frac{100\sqrt{6}}{3}h - 10000 = 0$, $h = 47.8$ or 或 -78.4 (rejected 捨去)

(b) $QS = 27.6$ m, $ST = 27.6 \times \sin 45° = 19.5$ m

(c) $\tan \angle PTS = \frac{47.8}{19.5}$, $\angle PTS = 67.8°$

5-1

1. (a) $5(6) - 4m = 20$, $m = \frac{10}{4} = \frac{5}{2}$ (b) $5(m) + 4(5) = 0$, $m = -4$

   (c) $m = 4$ (d) $m = -5$

2. (a) $5x - 6y = 5(0) - 6(1) = -6 \neq -7$, $\therefore$ no 不是

   (b) $3y - 4x = 3(2) - 4(0) = 6$, $\therefore$ yes 是

   (c) $2x + 3y = 1 + 3(\frac{1}{3}) = 2$, $\therefore$ yes 是

   (d) $5y - 2 = 5(-3) - 2 = -17 \neq 4(8)$, $\therefore$ no 不是

5-2

1. $x = 7y - 50$ --- (1); $5x + 6y = 37$ --- (2); Sub.代 (1) into 入 (2),
   $\therefore$ $5(7y - 50) + 6y = 37$, $35y - 250 + 6y = 37$, $y = 7$; $x = -1$; $\therefore$ $(-1, 7)$
2. $x = 2y - 9$ --- (1); $8x = 3y + 32$ --- (2); Sub.代 (1) into 入 (2),
   $\therefore$ $8(2y - 9) = 3y + 32$, $16y - 72 = 3y + 32$, $y = 8$; $x = 7$; $\therefore$ $(7, 8)$
3. $3x + 5y = -12$ --- (1); $x = -6 - y$ --- (2); Sub.代 (2) into 入 (1),
   $\therefore$ $3(-6 - y) + 5y = -12$, $-18 - 3y + 5y = -12$, $y = 3$; $x = -9$; $\therefore$ $(-9, 3)$
4. $3x + 4y = 14$ --- (1); $y = 4 - x$ --- (2); Sub.代 (2) into 入 (1),
   $\therefore$ $3x + 4(4 - x) = 14$, $3x + 16 - 4x = 14$, $x = 2$; $y = 2$; $\therefore$ $(2, 2)$
5. $3x + 8y = -22$ --- (1); $-3x + 8y = -10$ --- (2); (1) + (2),
   $\therefore$ $16y = -32$, $y = -2$, $x = -2$; $\therefore$ $(-2, -2)$
6. $3y = -7 - 9x$ --- (1); $3y = 4x + 9$ --- (2); Sub.代 (1) into 入 (2),
   $\therefore$ $-7 - 9x = 4x + 9$, $x = -\frac{16}{13}$; $y = \frac{53}{39}$; $\therefore$ $(-\frac{16}{13}, \frac{53}{39})$
7. $-5x + 7y = 4$ --- (1); $x = y$ --- (2); Sub.代 (2) into 入 (1),
   $\therefore$ $-5y + 7y = 4$, $y = 2$; $x = 2$; $\therefore$ $(2, 2)$
8. $x = -y - 3$ --- (1); $5x = 4y - 6$ --- (2); Sub.代 (1) into 入 (2),
   $\therefore$ $5(-y - 3) = 4y - 6$, $-5y - 15 = 4y - 6$, $y = -1$; $x = -2$; $\therefore$ $(-2, -1)$
9. $5x - 8y = -6$ --- (1); $3x - 4y = 6$ --- (2); (2)×2, $6x - 8y = 12$ --- (3)
   (1) – (3), $\therefore$ $x = 18$, $y = 12$; $\therefore (18, 12)$
10. $3y = 3x + 20$ --- (1); $y = \frac{5x}{2}$ --- (2); Sub.代 (2) into 入 (1),
    $\therefore$ $3(\frac{5x}{2}) = 3x + 20$, $15x = 6x + 40$, $x = \frac{40}{9}$; $y = \frac{100}{9}$; $\therefore$ $(\frac{40}{9}, \frac{100}{9})$

5-3

1. (a) $AB=\sqrt{(-8-(-2))^2+(3-11)^2}=\sqrt{100}=10$

   (b) $AB=\sqrt{(5-10)^2+(0-12)^2}=\sqrt{169}=13$

   (c) $AB=\sqrt{(-7-5)^2+(-4-(-4))^2}=\sqrt{144}=12$

2. (a) $AB=\sqrt{(\frac{3}{7}-3)^2+(5-\frac{5}{7})^2}=\sqrt{\frac{1224}{49}}=5.0$

   (b) $AB=\sqrt{(7-\frac{3}{5})^2+[-10-(-\frac{6}{5})]^2}=\sqrt{\frac{2960}{25}}=10.9$

   (c) $AB=\sqrt{(4-1)^2+(3-\frac{3}{10})^2}=\sqrt{\frac{1629}{100}}=4.0$

3. (a) $(-2-3)^2+(a-1)^2=13^2$; $(a-1)^2=144$; $a=13$ or 或 -11

   (b) $(2-8)^2+(3-k)^2=10^2$; $(3-k)^2=64$; $k=11$ or 或 -5

   (c) $(s-2)^2+(6-3)^2=5^2$; $(s-2)^2=16$; $s=6$ or 或 -2

4. $\sqrt{(-9-(-3))^2+(-3-5)^2}=\sqrt{(-9-1)^2+(-3-a)^2}$, $100=100+(a+3)^2$ $\therefore$ $a=-3$

5. $BC=AC$; $\sqrt{(-4-3)^2+(-1-a)^2}=\sqrt{(1-3)^2+(4-a)^2}$;

   $49+1+2a+a^2=4+16-8a+a^2$; $50+2a=20-8a$, $\therefore$ $a=-3$

6. $(-8-4)^2+(a-(-2))^2=15^2$; $144+a^2+4a+4=225$; $a^2+4a-77=0$;

   $a=7$ or 或 -11 (rejected 捨去), $\therefore$ $a=7$

7. $(3-a)^2+(2-(-4))^2=10^2$; $9-6a+a^2+36=100$; $a^2-6a-55=0$;

   $a=-5$ or 或 11 (rejected 捨去), $\therefore$ $a=-5$

5-4

1. (a) $(4,\frac{9}{2})$ (b) $(-\frac{1}{2},-\frac{11}{2})$ (c) $(8,-\frac{3}{2})$

2. (a) $=\left(\frac{(1)(-3)+3(9)}{1+3},\frac{(1)(5)+3(1)}{3+1}\right)=(6, 2)$

   (b) $=\left(\frac{2(9)+(1)(-6)}{3},\frac{2(13)+(7)(1)}{3}\right)=(4, 11)$

   (c) $=\left(\frac{(3)(1)+5(-7)}{3+5},\frac{3(-4)+5(17)}{3+5}\right)=(-4,\frac{73}{8})$

   (d) $=\left(\frac{3(15)+4(-6)}{7},\frac{3(9)+4(3)}{7}\right)=(3,\frac{39}{7})$

3. Let the coordinates of B be (x, y). 設 B 的座標為(x, y).

   (a) $-3=\frac{3(x)+4(-4)}{3+4}$, $x=-\frac{5}{3}$; $2=\frac{3(y)+4(5)}{3+4}$, $y=-2$

   (b) $12=\frac{(x)+4(2)}{5}$, $x=52$; $6=\frac{y+4(-9)}{5}$, $y=66$

   (c) $-3=\frac{7(x)+3(9)}{7+3}$, $x=-\frac{57}{7}$; $-6=\frac{7(y)+3(4)}{7+3}$, $y=-\frac{72}{7}$

4. Let 設 $AP : PB = r : s$.

(a) $\dfrac{r(7)+s(-3)}{r+s}=5$, $\dfrac{r}{s}=\dfrac{4}{1}$, $r:s=4:1$

(b) $\dfrac{r(-1)+s(2)}{r+s}=-5$, $\dfrac{r}{s}=-\dfrac{7}{4}$, $r:s=7:4$

(c) $\dfrac{r(\frac{1}{2})+s(-\frac{3}{2})}{r+s}=-\dfrac{7}{6}$, $\dfrac{r}{s}=\dfrac{1}{5}$, $r:s=1:5$

(d) $\dfrac{r(\frac{7}{3})+s(-\frac{2}{11})}{r+s}=\dfrac{69}{71}$, $\dfrac{r}{s}=\dfrac{3}{4}$, $r:s=3:4$

5-5

1. (a) $m=\dfrac{-8-(-6)}{4-5}=\dfrac{-2}{-1}=2$ (b) $m=\dfrac{7-2}{4-(-9)}=\dfrac{5}{13}$
   (c) $m=\dfrac{9-(-6)}{18-13}=\dfrac{15}{5}=3$ (d) $m=\dfrac{8-0}{-1-9}=\dfrac{-4}{5}$
   (e) $m=\dfrac{3-(-1)}{2-(-4)}=\dfrac{2}{3}$ (f) $m=\dfrac{-5-(-7)}{7-6}=2$
2. (a) $\dfrac{10-2}{7-a}=\dfrac{2}{7}$, $56=14-2a$, $a=-21$ (b) $\dfrac{-3-a}{-4-2}=-\dfrac{3}{2}$, $-6-2a=18$, $a=-12$
   (c) $\dfrac{1-a}{6+2}=-\dfrac{1}{6}$, $6-6a=-8$, $a=\dfrac{7}{3}$
3. (a) $\dfrac{5+10}{1+2}=\dfrac{y+10}{3+2}$, $5(5)=y+10$, $y=15$
   (b) $\dfrac{12+2}{4-x}=\dfrac{12-3}{4+2}$, $28=12-3x$, $x=-\dfrac{16}{3}$
4. (a) $m_{L_1}=-\dfrac{5}{7}$, $m_{L_2}=\dfrac{7}{5}$ (b) $m_{L_1}=\dfrac{1}{6}$, $m_{L_2}=-6$
   (c) $m_{L_1}=\dfrac{5}{2}$, $m_{L_2}=-\dfrac{2}{5}$ (d) $m_{L_1}=-\dfrac{3}{7}$, $m_{L_2}=\dfrac{7}{3}$
5. (a) $\dfrac{9-7}{6-2}=\dfrac{5+7}{x+4}$, $x+4=24$, $x=20$ (b) $\dfrac{3+9}{3+1}=\dfrac{y-1}{-3+2}$, $3=\dfrac{y-1}{-1}$, $y=-2$
   (c) $\dfrac{-4+7}{5-2}=\dfrac{5-4}{x+2}$, $x+2=1$, $x=-1$
6. (a) $\dfrac{-3+6}{10-5}\cdot\dfrac{y+19}{9-12}=-1$, $\dfrac{3}{5}\cdot\dfrac{y+19}{3}=-1$, $y=-14$
   (b) $\dfrac{4-(-4)}{3-1}\cdot\dfrac{9-(-5)}{x-8}=-1$, $4\cdot\dfrac{14}{x-8}=-1$, $x=-48$
   (c) $\dfrac{3-0}{3+6}\cdot\dfrac{2+5}{x-1}=-1$, $\dfrac{1}{3}\cdot\dfrac{7}{x-1}=-1$, $x=-\dfrac{4}{3}$

5-6

1. Horizontal 水平: (b), (e), (f); Vertical 垂直: (a); Neither 皆不是: (c), (d)
2. (a) $x=2$ (b) $y=-4$ (c) $x=-6$ (d) $y=5$
   (e) $y=x$ (f) $y=-x$

5-7

1. $\frac{y-1}{x-2}=3$, $y-1=3x-6$, $3x-y-5=0$
2. $\frac{y-7}{x+4}=-2$, $y-7=-2x-8$, $2x+y+1=0$
3. $\frac{y+3}{x-3}=-4$, $y+3=-4x+12$, $4x+y-9=0$
4. $\frac{y-1}{x-5}=\frac{5}{8}$, $8y-8=5x-25$, $5x-8y-17=0$
5. $\frac{y+2}{x+6}=-\frac{6}{7}$, $7y+14=-6x-36$, $6x+7y+50=0$
6. $\frac{y+3}{x-8}=\frac{2}{9}$, $9y+27=2x-16$, $2x-9y-43=0$
7. $\frac{y-6}{x+3}=\frac{1}{5}$, $5y-30=x+3$, $x-5y+33=0$
8. $\frac{y-2}{x+5}=-\frac{3}{10}$, $10y-20=-3x-15$, $3x+10y-5=0$
9. $\frac{y-2}{x+10}=\frac{4}{5}$, $5y-10=4x+40$, $4x-5y+50=0$
10. $\frac{y}{x}=-\frac{5}{2}$, $2y=-5x$, $5x+2y=0$
11. $\frac{y+3}{x+2}=-\frac{9}{10}$, $10y+30=-9x-18$, $9x+10y+48=0$
12. $\frac{y+4}{x-2}=\frac{1}{8}$, $8y+32=x-2$, $x-8y-34=0$
13. $\frac{y-1}{x+1}=\frac{17}{4}$, $4y-4=17x+17$, $17x-4y+21=0$
14. $\frac{y+\frac{13}{5}}{x-\frac{7}{12}}=2$, $y+\frac{13}{5}=2x-\frac{7}{6}$, $60x-30y-113=0$
15. $\frac{y-\frac{7}{11}}{x-\frac{5}{3}}=-1$, $y-\frac{7}{11}=-x+\frac{5}{3}$, $33x+33y-76=0$
16. $y+10=0$
17. $x-10=0$

5-8

1. $\frac{14-5}{2-11}=\frac{y-5}{x-11}$, $\frac{9}{-9}=\frac{y-5}{x-11}$, $x+y-16=0$
2. $\frac{5+2}{-4-5}=\frac{y+2}{x-5}$, $7(x-5)=-9(y+2)$, $7x+9y-17=0$
3. $\frac{5-3}{8-7}=\frac{y-3}{x-7}$, $2(x-7)=y-3$, $2x-y-11=0$
4. $\frac{9-1}{7+5}=\frac{y-1}{x+5}$, $2(x+5)=3(y-1)$, $2x-3y+13=0$

5. $\frac{8-4}{-1+7}=\frac{y-4}{x+7}$, $2(x+7)=3(y-14)$, $2x-3y+26=0$

6. $\frac{8-4}{1+5}=\frac{y-4}{x+5}$, $2(x+5)=3(y-4)$, $2x-3y+22=0$

7. $y-7=0$

8. $\frac{5-3}{9-7}=\frac{y-3}{x-7}$, $x-7=y-3$, $x-y-4=0$

9. $\frac{10+3}{9-2}=\frac{y+3}{x-2}$, $13(x-2)=7(y+3)$, $13x-7y-47=0$

10. $x+5=0$

11. $\frac{8-7}{5+7}=\frac{y-7}{x+7}$, $x+7=12(y-7)$, $x-12y+91=0$

12. $x-8=0$

13. $\frac{8-4}{0-6}=\frac{y-4}{x-6}$, $2(x-6)=-3(y-4)$, $2x+3y-24=0$

14. $\frac{0+4}{-5-1}=\frac{y+4}{x-1}$, $2(x-1)=-3(y+4)$, $2x+3y+10=0$

15. $\frac{y+\frac{7}{8}}{x-\frac{8}{7}}=\frac{\frac{4}{3}+\frac{7}{8}}{\frac{-3}{4}-\frac{8}{7}}$, $6(y+\frac{7}{8})=-7(x-\frac{8}{7})$, $28x+24y-11=0$

16. $\frac{\frac{11}{8}+\frac{8}{9}}{\frac{5}{6}-\frac{1}{12}}=\frac{y+\frac{8}{9}}{x-\frac{1}{12}}$, $163(36x-3)=54(36y+32)$,

$5868x-1944y-2217=0$, $1956x-648y-739=0$

5-9

1. $\frac{4-(-2)}{-8-6}=\frac{y+2}{x-6}$, $-\frac{3}{7}=\frac{y+2}{x-6}$, $3x+7y-4=0$

2. $\frac{9-(-8)}{1-(-3)}=\frac{y+8}{x+3}$, $\frac{17}{4}=\frac{y+8}{x+3}$, $17x-4y+19=0$

3. $x=-5$

4. $y=-4$

5. $\frac{0-(-4)}{-5-0}=\frac{y-(-4)}{x-0}$, $\frac{4}{-5}=\frac{y+4}{x}$, $4x+5y+20=0$

6. $\frac{5-0}{4-0}=\frac{y}{x}$, $5x=4y$, $5x-4y=0$

7. $\frac{10-0}{-2-0}=\frac{y}{x}$, $-5x=y$, $5x+y=0$

8. $\frac{0-(-6)}{8-0}=\frac{y+6}{x-0}$, $\frac{3}{4}=\frac{y+6}{x}$, $3x-4y-24=0$

9. $\frac{y-5}{x-8}=\frac{4}{3}$, $4x-3y-17=0$

10. $\frac{y-3}{x+8}=-3$, $y-3=-3x-24$, $3x+y+21=0$

5-10

1. $\frac{-x}{3}+\frac{y}{-9}=1$, $3x+y+9=0$

2. $\frac{3x}{7}+\frac{y}{4}=1$, $12x+7y-28=0$

3. $-5x+9y=-45$, $5x-9y-45=0$

4. $-7x+4y=-28$, $7x-4y-28=0$

5. $x-y=3$, $x-y-3=0$

6. $-2x+y=2$, $2x-y+2=0$

7. $7x-3y=7$, $x-\frac{3y}{7}=1$, $x-\frac{y}{\frac{7}{3}}=1$; x-int. (x 軸截距); y-int. (y 軸截距): $-\frac{7}{3}$

8. $8x-3y=-9$, $-\frac{8x}{9}+\frac{y}{3}=1$, $\frac{x}{-\frac{9}{8}}+\frac{y}{3}=1$; x-int. (x 軸截距): $-\frac{9}{8}$; y-int. (y 軸截距): 3

9. $9x-5y=-8$, $-\frac{9x}{8}+\frac{5y}{8}=1$, $\frac{x}{-\frac{8}{9}}+\frac{y}{\frac{8}{5}}=1$; x-int. (x 軸截距): $-\frac{8}{9}$; y-int. (y 軸截距): $\frac{8}{5}$

10. $7x-6y=42$, $\frac{x}{6}-\frac{y}{7}=1$; x-int. (x 軸截距): 6; y-int. (y 軸截距): -7

11. $9x+9y=-17$, $-\frac{9x}{17}-\frac{9y}{17}=1$, $\frac{x}{-\frac{17}{9}}-\frac{y}{\frac{17}{9}}=1$;

x-int. (x 軸截距): $-\frac{17}{9}$; y-int. (y 軸截距): $-\frac{17}{9}$

12. $7x-8y=10$, $\frac{7x}{10}-\frac{4y}{5}=1$, $\frac{x}{\frac{10}{7}}-\frac{y}{\frac{5}{4}}=1$; x-int. (x 軸截距): $\frac{10}{7}$; y-int. (y 軸截距): $-\frac{5}{4}$

13. $\frac{x}{4}+\frac{y}{3}=1$, $3x+4y-12=0$

14. $\frac{x}{-16}+\frac{y}{5}=1$, $5x-16y+80=0$

15. $\frac{x}{-8}+\frac{y}{-4}=1$, $x+2y+8=0$

16. $\frac{x}{a}+\frac{y}{-2a}=1$, $2x-y-2a=0$

5-11

1. (a) $y=4x+0$, $4x-y=0$
   (b) $y=-x+1$, $x+y-1=0$
   (c) $y=-\frac{2}{5}x+14$, $2x+5y-70=0$
   (d) $y=\frac{7}{4}x+(-4)$, $7x-4y-16=0$
   (e) $y=3x-\frac{9}{10}$, $30x-10y-9=0$
   (f) $y=-\frac{5}{3}x+2$, $5x+3y-6=0$

2. (a) $y=-\frac{x}{9}-\frac{5}{9}$; $m=-\frac{1}{9}$, $c=-\frac{5}{9}$
   (b) $y=x+\frac{2}{5}$; $m=1$, $c=\frac{2}{5}$
   (c) $y=9x+15$; $m=9$, $c=15$
   (d) $y=-\frac{4}{3}$; $m=0$, $c=-\frac{4}{3}$
   (e) $y=\frac{8}{5}x-\frac{7}{5}$; $m=\frac{8}{5}$, $c=-\frac{7}{5}$
   (f) $y=x-\frac{1}{4}$; $m=1$, $c=-\frac{1}{4}$
   (g) $x=\frac{5}{7}$; m: undefined 未下定義, c: no y-intercept 沒有 y 軸截距
   (h) $y=\frac{3}{5}x+\frac{8}{5}$; $m=\frac{3}{5}$, $c=\frac{8}{5}$
   (i) $y=\frac{6}{7}x$; $m=\frac{6}{7}$, $c=0$

3. (a) $m_1=-\frac{2}{3}$, $m_2=\frac{3}{2}$, $\perp$
   (b) $m_1=\frac{3}{7}$, $m_2=\frac{3}{2}$, neither 都不是
   (c) $m_1=\frac{3}{7}$, $m_2=\frac{3}{7}$, //
   (d) $m_1=-\frac{8}{5}$, $m_2=-\frac{8}{5}$, //
   (e) $m_1=4$, $m_2=-\frac{7}{5}$, neither 都不是
   (f) $m_1=\infty$, $m_2=0$, $\perp$

(g) $m_1 = \frac{9}{4}$, $m_2 = \frac{9}{4}$, //

(h) $m_1 = -\frac{1}{7}$, $m_2 = \frac{1}{7}$, //

5-12

1. $m_L = \frac{8}{6} = \frac{4}{3}$, $y = \frac{4}{3}x + 7$, $4x - 3y + 21 = 0$
2. $m_L = \frac{5}{7}$, $y = \frac{5}{7}x + 5$, $5x - 7y + 35 = 0$
3. $m_L = \frac{6}{15} = \frac{2}{5}$, $y = \frac{2}{5}x - 25$, $2x - 5y - 125 = 0$
4. $m_L = \frac{7}{9}$, $y = \frac{7}{9}x - 4$, $7x - 9y - 36 = 0$
5. $m_L = \frac{2}{5}$, $y = \frac{2}{5}x + 14$, $2x - 5y + 70 = 0$
6. $m_L = \frac{6}{9} = \frac{2}{3}$, $y = \frac{2}{3}x - \frac{9}{7}$, $14x - 21y - 27 = 0$
7. $m_L = \frac{5}{7}$, $\frac{y-13}{x-8} = \frac{5}{7}$, $5x - 7y + 51 = 0$
8. m_L: undefined 未下定義, $x - 6 = 0$
9. $m_L = \frac{1}{8}$, $\frac{y-3}{x+7} = \frac{1}{8}$, $x - 8y + 31 = 0$
10. $m_L = -\frac{3}{4}$, $\frac{y-9}{x+8} = -\frac{3}{4}$, $3x + 4y - 12 = 0$
11. $m_L = \frac{6}{7}$, $\frac{y-2}{x+6} = \frac{6}{7}$, $6x - 7y + 50 = 0$
12. $m_L = \frac{1}{3}$, $\frac{x-1}{x-4} = \frac{1}{3}$, $x - 3y - 1 = 0$

5-13

1. $m_{L'} = 2$, $m_L = -\frac{1}{2}$
2. $m_{L'} = \frac{9}{7}$, $m_L = -\frac{7}{9}$
3. $m_{L'} = \frac{7}{8}$, $m_L = -\frac{8}{7}$
4. $m_{L'} = 7$, $m_L = -\frac{1}{7}$
5. $m_{L'} = -2$, $m_L = \frac{1}{2}$
6. $m_{L'} = 0$, m_L: undefined 未下定義
7. $m = -1$, $\frac{y-2}{x-1} = -1$, $x + y - 3 = 0$
8. $m = -\frac{1}{7}$, $\frac{y+8}{x-2} = -\frac{1}{7}$, $x + 7y + 54 = 0$
9. $m = \frac{1}{4}$, $\frac{y+8}{x+7} = \frac{1}{4}$, $x - 4y - 25 = 0$
10. $m = -1$, $\frac{y-2}{x+6} = -1$, $x + y + 4 = 0$
11. $m = -18$, $\frac{y+4}{x-14} = -18$, $18x + y - 248 = 0$
12. $m = -\frac{1}{5}$, $\frac{y+2}{x-8} = -\frac{1}{5}$, $x + 5y + 2 = 0$
13. m: undefined 未下定義, $x + 8 = 0$
14. $m = 0$, $y - 3 = 0$
15. $m = -2$, $\frac{y-9}{x-19} = -2$, $2x + y - 47 = 0$

5-14

1. $m_L = m_{L_1} = \dfrac{6+7}{10-0} = \dfrac{13}{10}$, $\dfrac{y-0}{x+3} = \dfrac{13}{10}$, $13x - 10y + 39 = 0$
2. $m_L = -1 \div (-\dfrac{4}{3}) = \dfrac{3}{4}$, $\dfrac{y-0}{x+5} = \dfrac{3}{4}$, $3x - 4y + 15 = 0$
3. $m_L = -1 \div \dfrac{4}{5} = -\dfrac{5}{4}$, $\dfrac{y}{x} = -\dfrac{5}{4}$, $5x + 4y = 0$
4. $m_L = m_{L_1} = \dfrac{6+3}{6-4} = \dfrac{9}{2}$, $\dfrac{y+2}{x+10} = \dfrac{9}{2}$, $9x - 2y + 86 = 0$
5. $m_{L_1} = \dfrac{6-0}{0-10} = -\dfrac{3}{5}$, $m_L = -1 \div (-\dfrac{3}{5}) = \dfrac{5}{3}$, $\dfrac{y-0}{x-0} = \dfrac{5}{3}$, $5x - 3y = 0$
6. $m_{L_1} = \dfrac{1}{6}$, $m_L = -1 \div \dfrac{1}{6} = -6$, $\dfrac{y-3}{x+12} = -6$, $6x + y + 69 = 0$
7. $m_L = m_{L_1} = \dfrac{9-4}{-8-0} = -\dfrac{5}{8}$, $\dfrac{y-0}{x+10} = -\dfrac{5}{8}$, $5x + 8y + 50 = 0$
8. $m_L = m_{L_1} = \dfrac{0+8}{-3-0} = \dfrac{8}{-3}$, $\dfrac{y-0}{x-1} = -\dfrac{8}{3}$, $8x + 3y - 8 = 0$

5-15

1. $M(2, \dfrac{17}{2})$, $m_{PQ} = \dfrac{1}{10}$, $m_L = -10$, $\dfrac{y - \frac{17}{2}}{x-2} = -10$, $20x + 2y - 57 = 0$
2. $M(-1, 6)$, $m_{PQ} = 2$, $m_L = -\dfrac{1}{2}$, $\dfrac{y-6}{x+1} = -\dfrac{1}{2}$, $x + 2y - 11 = 0$
3. $M(-\dfrac{15}{2}, 4)$, $m_{PQ} = -\dfrac{2}{3}$, $m_L = \dfrac{3}{2}$, $\dfrac{y-4}{x + \frac{15}{2}} = \dfrac{3}{2}$, $6x - 4y + 61 = 0$
4. $M(\dfrac{11}{2}, -\dfrac{11}{2})$, $m_{PQ} = \dfrac{-2+9}{6-5} = 7$, $m_L = -\dfrac{1}{7}$, $\dfrac{y + \frac{11}{2}}{x - \frac{11}{2}} = -\dfrac{1}{7}$, $x + 7y + 33 = 0$
5. $M(15, 1)$, $m_{PQ} = \dfrac{3}{8}$, $m_L = -\dfrac{8}{3}$, $\dfrac{y-1}{x-15} = -\dfrac{8}{3}$, $8x + 3y - 123 = 0$
6. $M(7, 1)$, $m_{PQ} = -\dfrac{7}{2}$, $m_L = \dfrac{2}{7}$, $\dfrac{y-1}{x-7} = \dfrac{2}{7}$, $2x - 7y - 7 = 0$

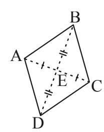
5-16

1. $M(-7, \dfrac{15}{2})$, $\dfrac{y-1}{x-2} = \dfrac{\frac{15}{2} - 1}{-7-2}$, $13x + 18y - 44 = 0$
2. $\dfrac{y-5}{x+7} = \dfrac{2+6}{10+4}$, $4x - 7y + 63 = 0$
3. $M(5, 5)$, $\dfrac{y-5}{x-5} = \dfrac{1}{3}$, $x - 3y + 10 = 0$
4. Mid-point of BD 的中點 $= (-1, 0)$, $m_{BD} = \dfrac{1+1}{-4-2} = -\dfrac{1}{3}$, $m_{AC} = 3$, $\dfrac{y-0}{x+1} = 3$,
   $3x - y + 3 = 0$

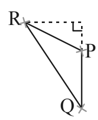
5. Mid-point of AB 的中點 $=(-4,\frac{3}{2})$, $\frac{y-2}{x-7}=\frac{\frac{3}{2}-2}{-4-7}$, $x-22y+37=0$

5-17

1. $AP=AQ$, $(4-2)^2+(-1-3)^2=(4-0)^2+(-1-b)^2$, $-1-b=2$ or 或 -2, $\therefore b=-3$ or 或 1
2. $[4-(2-k)]^2+[(1+k)-(k-1)]^2=13$, $2+k=3$ or 或 -3, $\therefore k=1$ or 或 -5
3. $m_{AB}=m_{BC}$, $\frac{4-1}{-2-2}=\frac{y-1}{10-2}$, $y=-5$
4. $y=-\frac{2}{3}x-\frac{11}{3}$; $y=-\frac{3}{s}x-\frac{1}{s}$; $(-\frac{2}{3})\times(-\frac{3}{s})=-1$, $s=-2$
5. $y=\frac{1}{3}x-\frac{7}{3}$; $y=-\frac{m}{6}x-\frac{1}{3}$; $\frac{1}{3}=-\frac{m}{6}$, $m=-2$
6. $(8)+m(-1)-3=0$, $m=5$; $y=-\frac{1}{5}x+\frac{3}{5}$, slope 斜率 $=-\frac{1}{5}$
7. $E=(\frac{p+q}{2},\frac{q-p}{2})$. Let 設 $C=(x,y)$, $\frac{0+x}{2}=\frac{p+q}{2}$, $x=p+q$; $\frac{0+y}{2}=\frac{q-p}{2}$, $y=q-p$, $\therefore C=(p+q,q-p)$



8. Area 面積 $=\frac{1}{2}\times[-1-(-7)]\times[0-(-5)]=15$



9. $FG=\sqrt{(4-3)^2+(-2-0)^2}=\sqrt{5}$, $GH=\sqrt{(10-4)^2+[1-(-2)]^2}=3\sqrt{5}$, area 面積 $=\frac{\sqrt{5}\times3\sqrt{5}}{2}=7.5$
10. $2x+5y=0$, $y=-\frac{2}{5}x$; $m_L=\frac{t-(-3)}{-4-3}=-\frac{2}{5}$, $t=-\frac{1}{5}$
11. mid-point 中點 $=(\frac{-3+5}{2},\frac{8+(-2)}{2})=(1,3)$; $4x-3y+10=0$, $y=\frac{4}{3}x+\frac{10}{3}$; equation 方程: $\frac{y-3}{x-1}=-\frac{3}{4}$, $3x+4y-15=0$
12. $m_{BC}=\frac{2-0}{1-(-5)}=\frac{1}{3}$, equation 方程: $\frac{y-(-3)}{x-1}=-3$, $3x+y=0$
13. $\begin{cases}3x-y-3=0\\x+2y-8=0\end{cases}$, intersection point 交點 $=(2,3)$, $m_{L_1}=3$, equation 方程: $\frac{y-3}{x-2}=-\frac{1}{3}$, $x+3y-11=0$

5-18

1. (a) $\frac{y-4}{x-0}=\frac{-2-4}{-3-0}$, $2x-y+4=0$
   (b) Put 代入 $y=0$, x-intercept (x 軸截距) $=-2$

2. $D=(\frac{2(-3)+3(6)}{2+3},\frac{2(0)+3(18)}{2+3})=(2.4,10.8)$

$\therefore\ \frac{y-10.8}{x-2.4}=\frac{-2-10.8}{12-2.4}$, $4x+3y-42=0$

3. (a) Let 設 $BD:DA=r:s$, $\frac{r(-2)+s(5)}{r+s}=2$, $3s=4r$, $r:s=3:4$

(b) $\because$ ΔCBD, ΔCAD are of the same height (高度相等),

$\therefore$ ΔCBD area 面積: ΔCAD area 面積 $=BD:DA=3:4$,

ΔCAD area 面積 $=15\times\frac{4}{3}=20$

4. (a) Put 代入 $y=0$, $x=10$, $M=(10,0)$. Put 代入 $x=0$, $y=-6$, $N=(0,-6)$

(b) $P=(\frac{10+0}{2},\frac{0+(-6)}{2})=(5,-3)$, $m_{OP}=\frac{-3-0}{5-0}=-\frac{3}{5}$

5. (a) $\frac{y-(-4)}{x-(-7)}=\frac{12}{5}$, $12x-5y+64=0$

(b) $12(-2)-5(b)+64=0$, $b=8$;

$AC=\sqrt{[-2-(-7)]^2+[8-(-4)]^2}=13$, $\therefore$ $AB=13$, $a=-7+13=6$

(c) $BC=\sqrt{[6-(-2)]^2+(-4-8)^2}=4\sqrt{13}$, perimeter 周界 $=13\times2+4\sqrt{13}=40.4$

6. (a) $y=-\frac{3}{2}x+6$, $m_{L_1}=-\frac{3}{2}$ (b) Put 代入 $y=0$, $x=4$, $C=(4,0)$

(c) $\frac{y-0}{x-4}=\frac{2}{3}$, $2x-3y-8=0$

(d) $A=(0,6)$, $B=(0,-\frac{8}{3})$, area 面積 $=\frac{1}{2}[6-(-\frac{8}{3})](4)=\frac{52}{3}$

7. (a) $m_{PR}=\frac{1-(-3)}{6-0}=\frac{2}{3}$; $m_{PS}=\frac{-4-(-3)}{5-0}=-\frac{1}{5}$; $m_{RS}=\frac{1-(-4)}{6-5}=5$

equation of PR (PR 的方程): $\frac{y-(-3)}{x-0}=\frac{2}{3}$, $2x-3y-9=0$

equation of PS (PS 的方程): $\frac{y-(-3)}{x-0}=-\frac{1}{5}$, $x+5y+15=0$

equation of RS (RS 的方程): $\frac{y-(-4)}{x-5}=5$, $5x-y-29=0$

(c) $\because QS\perp PR$, $\therefore m_{QS}=-1\div\frac{2}{3}=-\frac{3}{2}$, equation 方程: $\frac{y-(-4)}{x-5}=-\frac{3}{2}$, $3x+2y-7=0$

8. (a) $m_{PQ}=\frac{9-0}{0-(-4)}=\frac{9}{4}$ (b) $\frac{y-0}{x-16}=\frac{12-0}{-11-16}$, $4x+9y-64=0$

(c) $m_{RS}=\frac{12-0}{-11-16}=-\frac{4}{9}$, $m_{PQ}\times m_{RS}=\frac{9}{4}\times(-\frac{4}{9})=-1$, $\therefore$ $RS\perp PQ$.

$\because$ Orthocenter lies on altitudes 垂心是在高線之上的, $\therefore$ Yes 是

In Unit 6, r represents radius unless otherwise stated.
於第 6 課，除非特別註明，否則 r 表示半徑。

6-1

1. $(x-3)^2+(y-1)^2=2^2$, $x^2+y^2-6x-2y+6=0$

2. $(x+5)^2+(y-3)^2=4^2$, $x^2+y^2+10x-6y+18=0$
3. $(x-4)^2+(y+2)^2=3^2$, $x^2+y^2-8x+4y+11=0$
4. $(x+1)^2+(y+7)^2=1.5^2$, $x^2+y^2+2x+14y+47.75=0$
5. $(x-\frac{1}{2})^2+(y+\frac{3}{2})^2=(\sqrt{5})^2$, $x^2+y^2-x+3y-\frac{5}{2}=0$
6. $(x-0)^2+(y+5)^2=[1-(-5)]^2$, $x^2+y^2+10y-11=0$
7. $(x-3)^2+(y+2)^2=4^2$, $x^2+y^2-6x+4y-3=0$
8. $(x+2)^2+(y-0)^2=6^2$, $x^2+y^2+4x-32=0$
9. $(x-8)^2+(y-12)^2=9^2$, $x^2+y^2-16x-24y+127=0$
10. $(x+\frac{3}{2})^2+(y+3)^2=(\frac{5}{4})^2$, $x^2+y^2+3x+6y+\frac{155}{16}=0$
11. $(x-5.5)^2+(y+0.5)^2=2.5^2$, $x^2+y^2-11x+y+24.25=0$
12. $(x-a)^2+(y+a)^2=(\frac{\sqrt{3}}{2})^2$, $x^2+y^2-2ax+2ay+2a^2-\frac{3}{4}=0$

6-2

1. $(0,0); 2$
2. $(0,1); 4$
3. $(3,5); 5$
4. $(-5,8); 11$
5. $(-1.5,-4); \sqrt{3}$
6. $(-16,9); \sqrt{10}$
7. $(3,0)$; $r=\sqrt{3^2+0^2-0}=3$
8. $(0,4)$; $r=\sqrt{0^2+4^2-12}=2$
9. $(-2,2)$; $r=\sqrt{(-2)^2+2^2-(-28)}=6$
10. $(-1,-5)$; $r=\sqrt{1^2+5^2-17}=3$
11. $(1.5,-0.5)$; $r=\sqrt{1.5^2+0.5^2-(-1.5)}=2$
12. $\left(-\frac{1}{2},-\frac{1}{2}\right)$; $r=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(-6)}=\sqrt{\frac{13}{2}}$
13. $\left(-\frac{15}{2},1\right)$; $r=\sqrt{\left(\frac{15}{2}\right)^2+1^2-(-3)}=\frac{\sqrt{241}}{2}$
14. $\left(\frac{1}{6},-\frac{1}{3}\right)$; $r=\sqrt{\left(\frac{1}{6}\right)^2+\left(\frac{1}{3}\right)^2-\frac{1}{36}}=\frac{1}{3}$
15. $x^2+y^2-\frac{1}{2}x+\frac{7}{2}y-\frac{5}{2}=0$, center 圓心 $=\left(\frac{1}{4},-\frac{7}{4}\right)$; $r=\sqrt{\left(\frac{1}{4}\right)^2+\left(\frac{7}{4}\right)^2-\left(-\frac{5}{2}\right)}=\frac{3}{2}\sqrt{\frac{5}{2}}$
16. $x^2+y^2+\frac{8}{3}x-\frac{16}{3}y+\frac{7}{3}=0$, center 圓心 $=\left(-\frac{4}{3},\frac{8}{3}\right)$; $r=\sqrt{\left(\frac{4}{3}\right)^2+\left(\frac{8}{3}\right)^2-\frac{7}{3}}=\frac{\sqrt{59}}{3}$
17. $x^2+y^2+16x+32y-23=0$, center 圓心 $=(-8,-16)$; $r=\sqrt{8^2+16^2-(-23)}=7\sqrt{7}$
18. $x^2+y^2-30x+12y+18=0$, center 圓心 $=(15,-6)$; $r=\sqrt{15^2+6^2-18}=9\sqrt{3}$

6-3

1. center 圓心 $=(-7,1)$; $r=\sqrt{7^2+1^2-14}=6$

   $\sqrt{(-4+7)^2+(5-1)^2}=5<6$, $\therefore (-4,5)$ lies inside the circle 在圓內.

2. center 圓心 $=(4,1)$; $r=\sqrt{4^2+1^2+1}=\sqrt{18}$
$\sqrt{(7-4)^2+(-2-1)^2}=\sqrt{18}$, $\therefore (7,-2)$ lies on the circle 在圓上.
3. center 圓心 $=(-3,-3)$; $r=\sqrt{3^2+3^2+18}=6$
$\sqrt{(-3+3)^2+(-3-5)^2}=10>6$, $\therefore (3,5)$ lies outside the circle 在圓外.
4. center 圓心 $=(1,2)$; $r=\sqrt{1^2+2^2+24}=\sqrt{29}$
$\sqrt{(-3-1)^2+(-3-2)^2}=\sqrt{41}>\sqrt{29}$, $\therefore (-3,-3)$ lies outside the circle 在圓外.
5. center 圓心 $=(1,-5)$; $r=\sqrt{1^2+5^2-21}=\sqrt{5}$
$\sqrt{(3-1)^2+[-7-(-5)]^2}=\sqrt{8}>\sqrt{5}$, $\therefore (3,-7)$ lies outside the circle 在圓外.
6. center 圓心 $=(-4,-2)$; $r=\sqrt{4^2+2^2+17}=\sqrt{37}$
$\sqrt{(-4-1)^2+[-2-(-5)]^2}=\sqrt{34}<\sqrt{37}$, $\therefore (1,-5)$ lies inside the circle 在圓內.

6-4

1. center 圓心 $=A(5,0)$; $r=\sqrt{(5)^2+(0)^2-21}=2$, $AQ=\sqrt{(9-5)^2+(4-0)^2}=\sqrt{32}$
$PQ^2=AQ^2-r^2$ $(\because AP\perp PQ)$, $PQ=\sqrt{32-2^2}=2\sqrt{7}$

2. center 圓心 $=A(-3,-2)$; $r=\sqrt{(-3)^2+(-2)^2-(-3)}=4$,
$AQ=\sqrt{[1-(-3)]^2+[1-(-2)]^2}=5$, $PQ^2=AQ^2-r^2$ $(\because AP\perp PQ)$,
$PQ=\sqrt{5^2-4^2}=3$
3. center 圓心 $=A(8,-1)$; $r=\sqrt{(8)^2+(-1)^2-29}=6$, $AQ=-1-(-11)=10$
$PQ^2=AQ^2-r^2$ $(\because AP\perp PQ)$, $PQ=\sqrt{10^2-6^2}=8$
4. center 圓心 $=A(-6,2)$; $r=\sqrt{(-6)^2+(2)^2-20}=\sqrt{20}$,
$AQ=\sqrt{[2-(-6)]^2+(3-2)^2}=\sqrt{65}$, $PQ^2=AQ^2-r^2$ $(\because AP\perp PQ)$,
$PQ=\sqrt{65-20}=3\sqrt{5}$

6-5

1. $r=\sqrt{(5-2)^2+[3-(-1)]^2}=5$, $\therefore$ $(x-2)^2+(y+1)^2=25$
2. $r=\sqrt{(5-4)^2+[-3-(-6)]^2}=\sqrt{10}$, $\therefore$ $(x-4)^2+(y+6)^2=10$
3. $r=\sqrt{(-5-0)^2+(-1-0)^2}=\sqrt{26}$, $\therefore$ $(x+5)^2+(y+1)^2=26$
4. $r=\sqrt{(-3-0)^2+(-1-3)^2}=5$, $\therefore$ $(x+3)^2+(y+1)^2=25$
5. $r=\sqrt{[-1-(-9)]^2+(-2-4)^2}=10$, $\therefore$ $(x+9)^2+(y-4)^2=100$
6. $r=\sqrt{(-8-4)^2+(4-9)^2}=13$, $\therefore$ $(x-4)^2+(y-9)^2=169$
7. $r=\sqrt{(7-2)^2+[-5-(-6)]^2}=\sqrt{26}$, $\therefore$ $(x-2)^2+(y+6)^2=26$
8. $r=\sqrt{[-4-(-1)]^2+(-1-5)^2}=\sqrt{45}$, $\therefore$ $(x+1)^2+(y-5)^2=45$

9. $r=\sqrt{\left[3-\left(-\frac{1}{2}\right)\right]^2+\left(-\frac{1}{2}-4\right)^2}=\sqrt{\frac{49}{4}+\frac{81}{4}}=\sqrt{\frac{65}{2}}$ $\therefore$ $(x-3)^2+\left(y+\frac{1}{2}\right)^2=\frac{65}{2}$

10. $r=\sqrt{\left[-\frac{1}{3}-(-1)\right]^2+\left(2-\frac{1}{2}\right)^2}=\sqrt{\frac{4}{9}+\frac{9}{4}}=\frac{\sqrt{97}}{6}$, $\therefore$ $\left(x+\frac{1}{3}\right)^2+(y-2)^2=\frac{97}{36}$

6-6

1. $r=3$; $(x-3)^2+y^2=9$
2. $r=8$; $(x+2)^2+(y-8)^2=64$
3. $r=5$; $(x-6)^2+(y+5)^2=25$
4. $r=2$; $(x+2)^2+(y-4)^2=4$
5. $r=7$; $(x+7)^2+(y+3)^2=49$
6. $r=1.5$; $(x+1)^2+(y+1.5)^2=2.25$
7. $r=8$; $(x-1)^2+(y+8)^2=64$
8. $r=2$; $(x-2)^2+(y-7)^2=4$
9. $r=5$; $(x+5)^2+(y-6)^2=25$
10. $r=4$; $(x+3)^2+(y+4)^2=16$

6-7

1. center 圓心 $=\left(\frac{1+9}{2},\frac{1+7}{2}\right)=(5,4)$; $r=\sqrt{(9-5)^2+(7-4)^2}=5$,
 $\therefore$ $(x-5)^2+(y-4)^2=25$, $x^2+y^2-10x-8y+16=0$
2. center 圓心 $=\left(\frac{-4+8}{2},\frac{4-12}{2}\right)=(2,-4)$; $r=\sqrt{(-4-2)^2+[4-(-4)]^2}=10$
 $\therefore$ $(x-2)^2+(y+4)^2=100$, $x^2+y^2-4x+8y-80=0$
3. center 圓心 $=\left(\frac{2-2}{2},\frac{-1+3}{2}\right)=(0,1)$; $r=\sqrt{(2-0)^2+(-1-1)^2}=2\sqrt{2}$
 $\therefore$ $x^2+(y-1)^2=8$, $x^2+y^2-2y-7=0$
4. center 圓心 $=\left(\frac{-2-4}{2},\frac{-7+3}{2}\right)=(-3,-2)$; $r=\sqrt{(-3+4)^2+(-2-3)^2}=\sqrt{26}$
 $\therefore$ $(x+3)^2+(y+2)^2=26$, $x^2+y^2+6x+4y-13=0$
5. center 圓心 $=\left(\frac{1-2}{2},\frac{6+2}{2}\right)=\left(-\frac{1}{2},4\right)$; $r=\sqrt{\left(-\frac{1}{2}-1\right)^2+(4-6)^2}=\frac{5}{2}$
 $\therefore$ $\left(x+\frac{1}{2}\right)^2+(y-4)^2=\frac{25}{4}$, $x^2+y^2+x-8y+10=0$
6. center 圓心 $=\left(\frac{-5+4}{2},\frac{-3+0}{2}\right)=\left(-\frac{1}{2},-\frac{3}{2}\right)$; $r=\sqrt{\left(-\frac{1}{2}-4\right)^2+\left(-\frac{3}{2}-0\right)^2}=\sqrt{\frac{45}{2}}$
 $\therefore$ $\left(x+\frac{1}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{45}{2}$, $x^2+y^2+x+3y-20=0$
7. $\frac{y-1}{x-1}\times\frac{y-(-4)}{x-(-5)}=-1$, $y^2+3y-4=-(x^2+4x-5)$, $x^2+y^2+4x+3y-9=0$
8. $\frac{y-2}{x-(-2)}\times\frac{y-5}{x-6}=-1$, $y^2-7y+10=-(x^2-4x-12)$, $x^2+y^2-4x-7y-2=0$
9. $\frac{y-3}{x-(-3)}\times\frac{y-(-6)}{x-1}=-1$, $y^2+3y-18=-(x^2+2x-3)$, $x^2+y^2+2x+3y-21=0$

10. $\dfrac{y-4}{x-1}\times\dfrac{y-(-1)}{x-(-8)}=-1$, $y^2-3y-4=-(x^2+7x-8)$, $x^2+y^2+7x-3y-12=0$

11. $\dfrac{y-(-3)}{x-2}\times\dfrac{y-(-2)}{x-10}=-1$, $y^2+5y+6=-(x^2-12x+20)$, $x^2+y^2-12x+5y+26=0$

12. $\dfrac{y-4}{x-1.5}\times\dfrac{y-2}{x-3.5}=-1$, $y^2-6y+8=-(x^2-5x+5.25)$, $x^2+y^2-5x-6y+13.25=0$

6-8

1. $(0)^2+(0)^2+D(0)+E(0)+F=0$, $F=0$
$(2)^2+(-4)^2+D(2)+E(-4)+F=0$, $D-2E=-10$ ……(1)
$(-4)^2+(-2)^2+D(-4)+E(-2)+F=0$, $2D+E=10$ ……(2)
$\therefore D=2, E=6, F=0$
$\therefore$ $x^2+y^2+2x+6y=0$; center 圓心 $=(-1,-3)$; $r=\sqrt{1^2+3^2-0}=\sqrt{10}$

2. $(0)^2+(0)^2+D(0)+E(0)+F=0$, $F=0$
$(2)^2+(6)^2+D(2)+E(6)+F=0$, $D+3E=-20$ ……(1)
$(-4)^2+(8)^2+D(-4)+E(8)+F=0$, $D-2E=20$ ……(2)
$\therefore D=4, E=-8, F=0$
$\therefore x^2+y^2+4x-8y=0$; center 圓心 $=(-2,4)$; $r=\sqrt{2^2+4^2-0}=2\sqrt{5}$

3. $(1)^2+(1)^2+D(1)+E(1)+F=0$, $D+E+F=-2$ ……(1)
$(0)^2+(-1)^2+D(0)+E(-1)+F=0$, $-E+F=-1$ ……(2)
$(-5)^2+(4)^2+D(-5)+E(4)+F=0$, $-5D+4E+F=-41$ ……(3)
$\therefore D=5, E=-3, F=-4$
$\therefore x^2+y^2+5x-3y-4=0$; center 圓心 $=\left(-\dfrac{5}{2},-\dfrac{3}{2}\right)$; $r=\sqrt{\left(\dfrac{5}{2}\right)^2+\left(\dfrac{3}{2}\right)^2-(-4)}=\dfrac{5}{\sqrt{2}}$

4. $(-2)^2+(1)^2+D(-2)+E(1)+F=0$, $-2D+E+F=-5$ ……(1)
$(4)^2+(1)^2+D(4)+E(1)+F=0$, $4D+E+F=-17$ ……(2)
$(-4)^2+(9)^2+D(-4)+E(9)+F=0$, $-4D+9E+F=-97$ ……(3)
$\therefore D=-2, E=-12, F=3$
$\therefore x^2+y^2-2x-12y+3=0$; center 圓心 $=(1,6)$; $r=\sqrt{1^2+6^2-3}=\sqrt{34}$

5. $(1)^2+(2)^2+D(1)+E(2)+F=0$, $D+2E+F=-5$ ……(1)
$(-3)^2+(-6)^2+D(-3)+E(-6)+F=0$, $-3D-6E+F=-45$ ……(2)
$(-7)^2+(2)^2+D(-7)+E(2)+F=0$, $-7D+2E+F=-53$ ……(3)
$\therefore D=6, E=2, F=-15$
$\therefore x^2+y^2+6x+2y-15=0$; center 圓心 $=(-3,-1)$; $r=\sqrt{3^2+1^2+15}=5$

6. $(-1)^2+(3)^2+D(-1)+E(3)+F=0$, $-D+3E+F=-10$ ……(1)
$(-9)^2+(1)^2+D(-9)+E(1)+F=0$, $-9D+E+F=-82$ ……(2)
$(-4)^2+(-2)^2+D(-4)+E(-2)+F=0$, $-4D-2E+F=-20$ ……(3)
$\therefore D=10, E=-4, F=12$
$\therefore x^2+y^2+10x-4y+12=0$; center 圓心 $=(-5,2)$; $r=\sqrt{5^2+2^2-12}=\sqrt{17}$

7. $(4)^2+(0)^2+D(4)+E(0)+F=0$, $4D+F=-16$ ……(1)

$(-6)^2+(-3)^2+D(-6)+E(-3)+F=0$, $\quad -6D-3E+F=-45 \quad \ldots\ldots(2)$

$\left(-\frac{5}{2}\right)^2+\left(\frac{7}{2}\right)^2+D\left(-\frac{5}{2}\right)+E\left(\frac{7}{2}\right)+F=0$, $\quad -5D+7E+2F=-37 \quad \ldots\ldots(3)$

$\therefore D=2, E=3, F=-24$

$\therefore x^2+y^2+2x+3y-24=0$; center 圓心 $=\left(-1,-\frac{3}{2}\right)$; $\quad r=\sqrt{1^2+\left(\frac{3}{2}\right)^2+24}=\frac{\sqrt{109}}{2}$

8. $\left(\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)^2+D\left(\frac{1}{2}\right)+E\left(-\frac{1}{2}\right)+F=0$, $\quad D-E+2F=-1 \quad \ldots\ldots(1)$

$(7)^2+(4)^2+D(7)+E(4)+F=0$, $\quad 7D+4E+F=-65 \quad \ldots\ldots(2)$

$(1)^2+(-4)^2+D(1)+E(-4)+F=0$, $\quad D-4E+F=-17 \quad \ldots\ldots(3)$

$\therefore D=-12, E=3, F=7$

$\therefore x^2+y^2-12x+3y+7=0$; center 圓心 $=\left(6,-\frac{3}{2}\right)$; $\quad r=\sqrt{6^2+\left(\frac{3}{2}\right)^2-7}=\frac{5\sqrt{5}}{2}$

6-9

1. centre 圓心 $A=(0,k)$, $\sqrt{(-3-0)^2+(-1-k)^2}=\sqrt{(6-0)^2+(2-k)^2}$, $\quad \therefore k=5$

$r=\sqrt{(6-0)^2+(2-5)^2}=\sqrt{45}$; $\quad \therefore \quad x^2+(y-5)^2=45$

2. centre 圓心 $A=(h,0)$, $\sqrt{(-2-h)^2+(4-0)^2}=\sqrt{(8-h)^2+(6-0)^2}$, $\quad \therefore h=4$

$r=\sqrt{(8-4)^2+(6-0)^2}=\sqrt{52}$; $\quad \therefore \quad (x-4)^2+y^2=52$

3. center 圓心 $A=(0,k)$, $\sqrt{(-2-0)^2+(-5-k)^2}=\sqrt{(3-0)^2+(-10-k)^2}$, $\quad \therefore k=-8$

$r=\sqrt{(3-0)^2+[-10-(-8)]^2}=\sqrt{13}$; $\quad \therefore \quad x^2+(y+8)^2=13$

4. centre 圓心 $A=(h,0)$, $\sqrt{(-6-h)^2+(2-0)^2}=\sqrt{(-3-h)^2+(-1-0)^2}$, $\quad \therefore h=-5$

$r=\sqrt{[-3-(-5)]^2+(-1-0)^2}=\sqrt{5}$; $\quad \therefore \quad (x+5)^2+y^2=5$

5. centre 圓心 $A=(h,k)$, $\quad h-k+4=0 \quad \ldots\ldots(1)$

$\sqrt{(-6-h)^2+(4-k)^2}=\sqrt{(h-0)^2+(k-6)^2}$, $\quad 3h+k+4=0 \quad \ldots\ldots(2)$

$\therefore h=-2, k=2$, $\quad r=\sqrt{(-2-0)^2+(2-6)^2}=\sqrt{20}$; $\quad \therefore \quad (x+2)^2+(y-2)^2=20$

6. center 圓心 $A=(h,k)$, $\quad h+2k+5=0 \quad \ldots\ldots(1)$

$\sqrt{(-6-h)^2+(3-k)^2}=\sqrt{(h-1)^2+(k-2)^2}$, $\quad 7h-k+20=0 \quad \ldots\ldots(2)$

$\therefore h=-3, k=-1$, $\quad r=\sqrt{(-3-1)^2+(-1-2)^2}=5$; $\quad \therefore \quad (x+3)^2+(y+1)^2=25$

7. center 圓心 $A=(h,k)$, $\quad 3h+k-8=0 \quad \ldots\ldots(1)$

$\sqrt{(-2-h)^2+(-2-k)^2}=\sqrt{(h-2)^2+(k-4)^2}$, $\quad 2h+3k-3=0 \quad \ldots\ldots(2)$

$\therefore h=3, k=-1$, $\quad r=\sqrt{(3-2)^2+(-1-4)^2}=\sqrt{26}$; $\quad \therefore \quad (x-3)^2+(y+1)^2=26$

8. center 圓心 $A=(h,k)$, $\quad 3h-2k-9=0 \quad \ldots\ldots(1)$

$\sqrt{(-3-h)^2+(-2-k)^2}=\sqrt{(h-3)^2+[k-(-8)]^2}$, $\quad h-k-5=0 \quad \ldots\ldots(2)$

$\therefore h=-1, k=-6$, $\quad r=\sqrt{(-1-3)^2+[-6-(-8)]^2}=\sqrt{20}$; $\quad \therefore \quad (x+1)^2+(y+6)^2=20$

6-10

1. put 代入 $x=0$,
   (a) $y^2-9y+18=0$, $y=3$ or 或 6, $\therefore (0,3),(0,6)$
   (b) $y^2+10y+25=0$, $y=-5$, $\therefore (0,-5)$
   (c) $2^2+(y-4)^2=5$, $(y-4)^2=1$, $y-4=1$ or 或 -1, $y=5$ or 或 3, $\therefore (0,5),(0,3)$
2. put 代入 $y=0$,
   (a) $2x^2-7x-4=0$, $x=-\frac{1}{2}$ or 或 4, $\therefore (-\frac{1}{2},0),(4,0)$
   (b) $(x+3)^2+5^2=25$, $(x+3)^2=0$, $x=-3$, $\therefore (-3,0)$
   (c) $3x^2-2x+4=0$, $\Delta=(-2)^2-4(3)(4)=-44<0$, $\therefore$ no x-intercept 沒有 x 軸截距
3. (a) put 代入 $y=0$, $x^2+Dx+4=0$, $\Delta=0$, $D^2-4(1)(4)=0$, $D=4$ or 或 -4
   (b) put 代入 $y=0$, $2x^2+Dx+18=0$, $\Delta=0$, $D^2-4(2)(18)=0$, $D=12$ or 或 -12
4. (a) put 代入 $x=0$, $y^2+Ey+9=0$, $\Delta=0$, $E^2-4(1)(9)=0$, $E=6$ or 或 $E=-6$
   (b) put 代入 $x=0$, $2y^2+Ey=0$, $\Delta=0$, $E^2-4(2)(0)=0$, $E=0$

6-11

1. $\begin{cases}x^2+y^2+2x-9=0\\x+y+3=0\end{cases}$, $x^2+(-x-3)^2+2x-9=0$, $x^2+4x=0$, $x=0$ or 或 $x=-4$
   When 當 $x=0$, $y=-3$; when 當 $x=-4$, $y=1$.
   $\therefore$ Points of intersection are 交點是 $(0,-3)$, $(-4,1)$.
2. $\begin{cases}x^2+y^2-6x-4y+9=0\\x-y-3=0\end{cases}$, $(y+3)^2+y^2-6(y+3)-4y+9=0$, $y^2-2y=0$,
   $y=0$ or 或 $y=2$. When 當 $y=0$, $x=3$; when 當 $y=2$, $x=5$.
   $\therefore$ Points of intersection are 交點是 $(3,0)$, $(5,2)$.
3. $\begin{cases}x^2+y^2-11x+4y+5=0\\x+y-2=0\end{cases}$, $x^2+(2-x)^2-11x+4(2-x)+5=0$, $2x^2-19x+17=0$,
   $x=1$ or 或 $x=\frac{17}{2}$. When 當 $x=1$, $y=1$; when 當 $x=\frac{17}{2}$, $y=-\frac{13}{2}$.
   $\therefore$ Points of intersection are 交點是 $(1,1)$, $(\frac{17}{2},-\frac{13}{2})$.
4. $\begin{cases}x^2+y^2+4x+4y+3=0\\x-2y-5=0\end{cases}$, $(2y+5)^2+y^2+4(2y+5)+4y+3=0$, $5y^2+32y+48=0$,
   $y=-4$ or 或 $y=-\frac{12}{5}$. When 當 $y=-4$, $x=-3$; when 當 $y=-\frac{12}{5}$, $x=\frac{1}{5}$.
   $\therefore$ Points of intersection are 交點是 $(-3,-4)$, $(\frac{1}{5},-\frac{12}{5})$.
5. $\begin{cases}x^2+y^2-2x+8y+7=0\\x+3y+1=0\end{cases}$, $(-3y-1)^2+y^2-2(-3y-1)+8y+7=0$, $y^2+2y+1=0$,
   $y=-1$. When 當 $y=-1$, $x=2$. $\therefore$ Point of intersection is 交點是 $(2,-1)$.
6. $\begin{cases}x^2+y^2-10x+6y+8=0\\3x-y+4=0\end{cases}$, $x^2+(3x+4)^2-10x+6(3x+4)+8=0$, $5x^2+16x+24=0$,

no solution. $\therefore$ No intersection points. 沒有交點。

7. $\begin{cases} x^2+y^2+2x-y-10=0 \\ 2x+y+6=0 \end{cases}$, $x^2+(-2x-6)^2+2x-(-2x-6)-10=0$, $5x^2+28x+40=0$,

$x=-4$ or 或 $x=-\frac{8}{5}$. When 當 $x=-4$, $y=2$; when 當 $x=-\frac{8}{5}$, $y=-\frac{14}{5}$.

$\therefore$ Points of intersection are 交點是 $(-4,2)$, $(-\frac{8}{5},-\frac{14}{5})$.

8. $\begin{cases} x^2+y^2+3x+8y+8=0 \\ 5x-4y+12=0 \end{cases}$, $x^2+(\frac{5x+12}{4})^2+3x+8(\frac{5x+12}{4})+8=0$, $x^2+8x+16=0$,

$x=-4$. When 當 $x=-4$, $y=-2$. $\therefore$ Point of intersection is 交點是 $(-4,-2)$.

9. $\begin{cases} x^2+y^2-15x-6y+16=0 \\ 2x+3y-1=0 \end{cases}$, $x^2+(\frac{1-2x}{3})^2-15x-6(\frac{1-2x}{3})+16=0$,

$13x^2-103x+127=0$, $x=\frac{103\pm\sqrt{4005}}{26}$, $x=7$ or 或 $x=\frac{16}{13}$.

When 當 $x=7$, $y=-4$; when 當 $x=\frac{16}{13}$, $y=-\frac{2}{13}$.

$\therefore$ Points of intersection are 交點是 $(7,-4)$, $(\frac{16}{13},-\frac{2}{13})$.

10. $\begin{cases} x^2+y^2-14x+2y+10=0 \\ 4x+3y+5=0 \end{cases}$, $x^2+(\frac{-4x-5}{3})^2-14x+2(\frac{-4x-5}{3})+10=0$,

$5x^2-22x+17=0$, $x=1$ or 或 $x=\frac{17}{5}$.

When 當 $x=1$, $y=-3$; when 當 $x=\frac{17}{5}$, $y=-\frac{31}{5}$.

$\therefore$ Points of intersection are 交點是 $(1,-3)$, $(\frac{17}{5},-\frac{31}{5})$.

6-12

1. $\begin{cases} x^2+y^2-2x-6y+1=0 \\ x-y=0 \end{cases}$, $x^2+x^2-2x-6x+1=0$, $2x^2-8x+1=0$

$\Delta=(-8)^2-4(2)(1)=56>0$. $\therefore$ Two intersection points. 兩個交點。

2. $\begin{cases} x^2+y^2+10x+4y+20=0 \\ x+y+2=0 \end{cases}$, $x^2+(-x-2)^2+10x+4(-x-2)+20=0$, $x^2+5x+8=0$

$\Delta=(5)^2-4(1)(8)=-7<0$. $\therefore$ No intersection points. 沒有交點。

3. $\begin{cases} x^2+y^2+6x-y+8=0 \\ 2x+y+3=0 \end{cases}$, $x^2+(-2x-3)^2+6x-(-2x-3)+8=0$, $x^2+4x+4=0$

$\Delta=(4)^2-4(1)(4)=0$. $\therefore$ One intersection point. 一個交點。

4. $\begin{cases} x^2+y^2-2x-2y-8=0 \\ x+2y+4=0 \end{cases}$, $(-2y-4)^2+y^2-2(-2y-4)-2y-8=0$, $5y^2+18y+16=0$

$\Delta=(18)^2-4(5)(16)=4>0$. $\therefore$ Two intersection points. 兩個交點。

5. $\begin{cases} x^2+y^2+8x-4y+15=0 \\ 3x-y+1=0 \end{cases}$, $x^2+(3x+1)^2+8x-4(3x+1)+15=0$, $5x^2+x+6=0$

$\Delta=(1)^2-4(5)(6)=-119<0$. $\therefore$ No intersection points. 沒有交點。

6. $\begin{cases} x^2+y^2+5x-10y-11=0 \\ 3x+5y+1=0 \end{cases}$, $x^2+(\frac{-3x-1}{5})^2+5x-10(\frac{-3x-1}{5})-11=0$,

$34x^2+281x-224=0$. $\Delta=(281)^2-4(34)(-224)=109425>0$.

$\therefore$ Two intersection points. 兩個交點。

7. $\begin{cases} x^2+y^2-11x+8y+17=0 \\ 3x-2y-5=0 \end{cases}$, $x^2+(\frac{3x-5}{2})^2-11x+8(\frac{3x-5}{2})+17=0$, $x^2-2x+1=0$

$\Delta=(-2)^2-4(1)(1)=0$. $\therefore$ One intersection point. 一個交點。

6-13

1. $\begin{cases} x^2+y^2+15x+5y+F=0 \\ y=-x+1 \end{cases}$, $x^2+(-x+1)^2+15x+5(-x+1)+F=0$,

$2x^2+8x+(6+F)=0$. $\Delta=0$, $8^2-4(2)(6+F)=0$, $F=2$

2. $\begin{cases} x^2+y^2+Dx-10y-19=0 \\ y=2x-3 \end{cases}$, $x^2+(2x-3)^2+Dx-10(2x-3)-19=0$,

$5x^2+(D-32)x+20=0$. $\Delta=0$, $(D-32)^2-4(5)(20)=0$, $D=12$ or 或 $D=52$

3. $\begin{cases} x^2+y^2+4x+Ey+27=0 \\ y=x+5 \end{cases}$, $x^2+(x+5)^2+4x+E(x+5)+27=0$,

$2x^2+(14+E)x+(5E+52)=0$.

$\Delta=0$, $(14+E)^2-4(2)(5E+52)=0$, $E=22$ or 或 $E=-10$

4. $\begin{cases} x^2+y^2+3x+7y+F=0 \\ x-4y=4 \end{cases}$, $(4y+4)^2+y^2+3(4y+4)+7y+F=0$,

$17y^2+51y+(28+F)=0$. $\Delta=0$, $(51)^2-4(17)(28+F)=0$, $F=10.25$

5. $\begin{cases} x^2+y^2+Dx+2y-40=0 \\ 3x+y+10=0 \end{cases}$, $x^2+(-3x-10)^2+Dx+2(-3x-10)-40=0$,

$10x^2+(D+54)x+40=0$. $\Delta=0$, $(D+54)^2-4(10)(40)=0$, $D=-14$ or 或 $D=-94$

6. $\begin{cases} x^2+y^2-2x+Ey-7=0 \\ 4x-3y+8=0 \end{cases}$, $x^2+(\frac{4x+8}{3})^2-2x+E(\frac{4x+8}{3})-7=0$,

$25x^2+(46+12E)x+(1+24E)=0$.

$\Delta=0$, $(46+12E)^2-4(25)(1+24E)=0$, $E=7$ or 或 $E=2$

7. $\begin{cases} x^2+y^2+12x-7y+F=0 \\ 2x+y+1=0 \end{cases}$, $x^2+(-2x-1)^2+12x-7(-2x-1)+F=0$,

$5x^2+30x+(8+F)=0$. $\Delta=0$, $(30)^2-4(5)(8+F)=0$, $F=37$

$\therefore 5x^2+30x+(8+37)=0$, $x=-3$. When 當 $x=-3$, $y=5$.

$\therefore$ The contact point is 切點是 $(-3,-5)$.

8. $\begin{cases} x^2+y^2-6x+Ey+5=0 \\ x-y+3=0 \end{cases}$, $x^2+(x+3)^2-6x+E(x+3)+5=0$, $2x^2+Ex+(14+3E)=0$

$\Delta = 0$, $E^2 - 4(2)(14+3E) = 0$, $E = 28$ or 或 $E = -4$.

When 當 $E = 28$, $2x^2 + 28x + 98 = 0$, $x = -7$, $y = -4$.

When 當 $E = -4$, $2x^2 - 4x + 2 = 0$, $x = 1$, $y = 4$.

$\therefore$ The contact point is 切點是 $(-7, -4)$ or 或 $(1, 4)$.

9. $\begin{cases} x^2 + y^2 + Dx + 10y - 14 = 0 \\ x - 3y + 4 = 0 \end{cases}$, $(3y-4)^2 + y^2 + D(3y-4) + 10y - 14 = 0$,

$10y^2 + (3D-14)y + (2-4D) = 0$.

$\Delta = 0$, $(3D-14)^2 - 4(10)(2-4D) = 0$, $D = -2$ or 或 $D = -\frac{58}{9}$.

When 當 $D = -2$, $10y^2 - 20y + 10 = 0$, $y = 1$, $x = -1$.

When 當 $D = -\frac{58}{9}$, $10y^2 - \frac{100}{3}y + \frac{250}{9} = 0$, $y = \frac{5}{3}$, $x = 1$.

$\therefore$ The contact point is 切點是 $(-1, 1)$ or 或 $(1, \frac{5}{3})$.

10. $\begin{cases} x^2 + y^2 + Dx - 4y + 17 = 0 \\ 3x - 2y + 4 = 0 \end{cases}$, $x^2 + (\frac{3x+4}{2})^2 + Dx - 4(\frac{3x+4}{2}) + 17 = 0$,

$13x^2 + 4Dx + 52 = 0$. $\Delta = 0$, $(4D)^2 - 4(13)(52) = 0$, $D = 13$ or 或 $D = -13$.

When 當 $D = 13$, $13x^2 + 52x + 52 = 0$, $x = -2$, $y = -1$.

When 當 $D = -13$, $13x^2 - 52x + 52 = 0$, $x = 2$, $y = 5$.

$\therefore$ The contact point is 切點是 $(-2, -1)$ or 或 $(2, 5)$.

6-14

1. $\begin{cases} x^2 + y^2 - 6x - 12y + F = 0 \\ x + y - 3 = 0 \end{cases}$, $x^2 + (3-x)^2 - 6x - 12(3-x) + F = 0$, $2x^2 + (F-27) = 0$

$\Delta = 0$, $(0)^2 - 4(2)(F-27) = 0$, $F = 27$.

$\therefore$ Equation of circle 圓的方程: $x^2 + y^2 - 6x - 12y + 27 = 0$

2. $\begin{cases} x^2 + y^2 - 10x + 2y + F = 0 \\ x - y - 2 = 0 \end{cases}$, $x^2 + (x-2)^2 - 10x + 2(x-2) + F = 0$, $2x^2 - 12x + F = 0$

$\Delta = 0$, $(-12)^2 - 4(2)(F) = 0$, $F = 18$.

$\therefore$ Equation of circle 圓的方程: $x^2 + y^2 - 10x + 2y + 18 = 0$

3. $\begin{cases} x^2 + y^2 + 4x + 2y + F = 0 \\ x + 3y - 5 = 0 \end{cases}$, $(5-3y)^2 + y^2 + 4(5-3y) + 2y + F = 0$,

$10y^2 - 40y + (45+F) = 0$. $\Delta = 0$, $(-40)^2 - 4(10)(45+F) = 0$, $F = -5$

$\therefore$ Equation of circle 圓的方程: $x^2 + y^2 + 4x + 2y - 5 = 0$

4. $\begin{cases} x^2 + y^2 - 4x + 6y + F = 0 \\ 2x + y + 7 = 0 \end{cases}$, $x^2 + (-2x-7)^2 - 4x + 6(-2x-7) + F = 0$,

$5x^2 + 12x + (7+F) = 0$. $\Delta = 0$, $(12)^2 - 4(5)(7+F) = 0$, $F = 0.2$

$\therefore$ Equation of circle 圓的方程: $x^2 + y^2 - 4x + 6y + 0.2 = 0$

5. $\begin{cases} x^2 + y^2 - 2x + 10y + F = 0 \\ 3x + 2y + 20 = 0 \end{cases}$, $x^2 + (\frac{-3x-20}{2})^2 - 2x + 10(\frac{-3x-20}{2}) + F = 0$,

$13x^2+52x+4F=0$. $\Delta=0$, $(52)^2-4(13)(4F)=0$, $F=13$

$\therefore$ Equation of circle 圓的方程: $x^2+y^2-2x+10y+13=0$

6. $\begin{cases} x^2+y^2+6x-6y+F=0 \\ -3x+7y-1=0 \end{cases}$, $(\frac{7y-1}{3})^2+y^2+6(\frac{7y-1}{3})-6y+F=0$,

$58y^2+58y+(9F-17)=0$. $\Delta=0$, $(58)^2-4(58)(9F-17)=0$, $F=3.5$

$\therefore$ Equation of circle 圓的方程: $x^2+y^2+6x-6y+3.5=0$

6-15

1. Sub 代入 $(1,2)$, L.H.S. $=(1)^2+(2)^2+4(1)-6(2)+3=0=$ R.H.S.

   $\therefore$ P lies on C. P 在 C 上。Center 圓心 $M=(-2,3)$;

   $m_{MP}=\frac{3-2}{-2-1}=-\frac{1}{3}$, $\therefore$ slope of tangent 切線的斜率 $=3$

   $\therefore$ Equation of tangent 切線的方程: $\frac{y-2}{x-1}=3$, $3x-y-1=0$

2. Sub 代入 $(3,7)$, L.H.S. $=(3)^2+(7)^2-10(3)-6(7)+14=0=$ R.H.S.

   $\therefore$ P lies on C. P 在 C 上。Center 圓心 $M=(5,3)$;

   $m_{MP}=\frac{7-3}{3-5}=-2$, $\therefore$ slope of tangent 切線的斜率 $=\frac{1}{2}$

   $\therefore$ Equation of tangent 切線的方程: $\frac{y-7}{x-3}=\frac{1}{2}$, $x-2y+11=0$

3. Sub 代入 $(-3,-1)$, L.H.S. $=(-3)^2+(-1)^2-2(-3)-4(-1)-20=0=$ R.H.S.

   $\therefore$ P lies on C. P 在 C 上。Center 圓心 $M=(1,2)$;

   $m_{MP}=\frac{2-(-1)}{1-(-3)}=\frac{3}{4}$, $\therefore$ slope of tangent 切線的斜率 $=-\frac{4}{3}$

   $\therefore$ Equation of tangent 切線的方程: $\frac{y-(-1)}{x-(-3)}=-\frac{4}{3}$, $4x+3y+15=0$

4. Sub 代入 $(4,2)$, L.H.S. $=(4)^2+(2)^2-6(4)+5(2)-6=0=$ R.H.S.

   $\therefore$ P lies on C. P 在 C 上。Center 圓心 $M=(3,-\frac{5}{2})$;

   $m_{MP}=\frac{2-(-\frac{5}{2})}{4-3}=\frac{9}{2}$, $\therefore$ slope of tangent 切線的斜率 $=-\frac{2}{9}$

   $\therefore$ Equation of tangent 切線的方程: $\frac{y-2}{x-4}=-\frac{2}{9}$, $2x+9y-26=0$

5. Sub 代入 $(1,-4)$, L.H.S. $=(1)^2+(-4)^2-8(1)+3(-4)+3=0=$ R.H.S.

   $\therefore$ P lies on C. P 在 C 上。Center 圓心 $M=(4,-\frac{3}{2})$;

   $m_{MP}=\frac{-\frac{3}{2}-(-4)}{4-1}=\frac{5}{6}$, $\therefore$ slope of tangent 切線的斜率 $=-\frac{6}{5}$

   $\therefore$ Equation of tangent 切線的方程: $\frac{y-(-4)}{x-1}=-\frac{6}{5}$, $6x+5y+14=0$

6. Sub 代入 $(\frac{1}{2},-\frac{9}{2})$, L.H.S. $=(\frac{1}{2})^2+(-\frac{9}{2})^2+5(\frac{1}{2})+2(-\frac{9}{2})-14=0=$ R.H.S.

$\therefore$ P lies on C. P 在 C 上。Center 圓心 $M=(-\frac{5}{2},-1)$;

$m_{MP}=\dfrac{-\frac{9}{2}-(-1)}{\frac{1}{2}-(-\frac{5}{2})}=-\frac{7}{6}$, $\therefore$ slope of tangent 切線的斜率 $=\frac{6}{7}$

$\therefore$ Equation of tangent 切線的方程: $\dfrac{y-(-\frac{9}{2})}{x-\frac{1}{2}}=\frac{6}{7}$, $12x-14y-69=0$

6-16

1. Equation of tangent 切線的方程: $y=mx$. $\begin{cases}x^2+y^2-12x-4y+20=0\\ y=mx\end{cases}$,

   $x^2+(mx)^2-12x-4(mx)+20=0$, $(1+m^2)x^2-(12+4m)x+20=0$

   $\Delta=0$, $(12+4m)^2-4(1+m^2)(20)=0$, $m=-\frac{1}{2}$ or 或 $m=2$

   $\therefore$ Equations of tangents 切線的方程: $y=-\frac{1}{2}x$ and 和 $y=2x$

2. Equation of tangent 切線的方程: $y-6=mx$. $\begin{cases}x^2+y^2-5x+3y-4=0\\ y=mx+6\end{cases}$,

   $x^2+(mx+6)^2-5x+3(mx+6)-4=0$, $(1+m^2)x^2+(15m-5)x+50=0$

   $\Delta=0$, $(15m-5)^2-4(1+m^2)(50)=0$, $m=7$ or 或 $m=-1$

   $\therefore$ Equations of tangents 切線的方程: $y=7x+6$ and 和 $y=-x+6$

3. Equation of tangent 切線的方程: $y=m(x-3)$. $\begin{cases}x^2+y^2+4x-10y+24=0\\ y=mx-3m\end{cases}$,

   $x^2+(mx-3m)^2+4x-10(mx-3m)+24=0$,

   $(1+m^2)x^2+(4-10m-6m^2)x+(9m^2+30m+24)=0$

   $\Delta=0$, $(4-10m-6m^2)^2-4(1+m^2)(9m^2+30m+24)=0$,

   $(9m^4+30m^3+13m^2-20m+4)-(9m^4+30m^3+33m^2+30m+24)=0$,

   $m=-2$ or 或 $m=-\frac{1}{2}$

   $\therefore$ Equations of tangents 切線的方程: $y=-2(x-3)$ and 和 $y=-\frac{1}{2}(x-3)$

   i.e. 即 $2x+y-6=0$ and 和 $x+2y-3=0$

6-17

1. centre 圓心 $=(2,-4)$, distance 距離 $=0-(-4)=4$
2. $r=\sqrt{(\frac{4}{2})^2+(\frac{14}{2})^2-F}=\sqrt{53-F}$; $r>0$, $53-F>0$, $F<53$
3. $r=\sqrt{(\frac{10}{2})^2+(\frac{-8}{2})^2-(-8)}=7$, $\therefore$ area 面積 $=\pi\times7^2=49\pi$
4. centre 圓心 $=(1,3)$; Let 設 $Q=(a,b)$. $\because$ Centre is the mid-pt of PQ (圓心是 PQ 的中點),

$\therefore \frac{a+3}{2}=1$, $a=-1$; $\frac{b+7}{2}=3$, $b=-1$, $\therefore (-1,-1)$

5. centre 圓心 $=(2,1)$, $\because y=3x+k$ is a diameter 是直徑, $\therefore 1=3(2)+k$, $k=-5$

6. put 代入 $(-1,10)$, $(-1)^2+10^2+k(-1)-8(10)-3=0$, $k=18$,

$r=\sqrt{(\frac{18}{2})^2+(\frac{-8}{2})^2-(-3)}=10$

7. When 當 $x=0$, $y^2-y-12=0$, $y=4,-3$; $\therefore$ length 長度 $=4-(-3)=7$

8. When 當 $y=0$, $x^2+4x-12=0$, $x=2,-6$; $\therefore$ length 長度 $=2-(-6)=8$

9. centre 圓心 $=(0,0)$; $\frac{-1-0}{3-0}=-\frac{1}{3}$, $\therefore$ slope of chord 弦的斜率 $=-1\div(-\frac{1}{3})=3$;

$\therefore \frac{y-(-1)}{x-3}=3$, $3x-y-10=0$

10. $L: 3x-4y=12$, $y=\frac{3}{4}x-3$, $\therefore m_L=\frac{3}{4}$; $\therefore \frac{s-0}{1-4}\times\frac{3}{4}=-1$, $s=4$

11. $r=3-(-1)=4$, $\therefore$ centre 圓心 $=(3,4)$ or 或 $(3,-4)$

$\therefore (x-3)^2+(y-4)^2=16$ or 或 $(x-3)^2+(y+4)^2=16$

12. $\begin{cases} x^2+y^2=4 \\ y=mx+6 \end{cases}$, $x^2+(mx+6)^2=4$, $(1+m^2)x^2+12mx+32=0$,

$\Delta=0$, $(12m)^2-4(1+m^2)(32)=0$, $m=\pm 2\sqrt{2}$

6-18

1. (a) $r_1=\sqrt{(\frac{2}{2})^2+(\frac{-4}{2})^2-(-15)}=2\sqrt{5}$; $r_2=\sqrt{(\frac{-18}{2})^2+(\frac{-14}{2})^2-85}=3\sqrt{5}$

(b) $O_1P:O_2P=2\sqrt{5}:3\sqrt{5}=2:3$

$O_1=(-1,2)$, $O_2=(9,7)$, $\therefore P=(\frac{2(9)+3(-1)}{2+3},\frac{2(7)+3(2)}{2+3})=(3,4)$

2. (a) centre 圓心 $=(2,-3)$; $r=\sqrt{2^2+(-3)^2-3}=\sqrt{10}$

(b) $OA=\sqrt{(11-2)^2+[0-(-3)]^2}=\sqrt{90}=3\sqrt{10}$

(c) shortest distance 最短距離 $=OA-r=3\sqrt{10}-\sqrt{10}=2\sqrt{10}$

3. (a) $\angle BCO=90°$ (semi-circle 半圓);

$\tan 30°=\frac{BC}{OC}$, $BC=OC\tan 30°=6\times\frac{1}{\sqrt{3}}=2\sqrt{3}$, $\therefore B=(6,2\sqrt{3})$

(b) A = mid-pt of OB (OB 的中點) $=(\frac{0+6}{2},\frac{0+2\sqrt{3}}{2})=(3,\sqrt{3})$

$r=\sqrt{(3-0)^2+(\sqrt{3}-0)^2}=\sqrt{12}=2\sqrt{3}$

(c) $(x-3)^2+(y-\sqrt{3})^2=12$

4. (a) $x^2+y^2-\frac{3}{2}x-2y-\frac{3}{2}=0$, $A=(\frac{3}{4},1)$; $r=\sqrt{(\frac{3}{4})^2+1^2-(-\frac{3}{2})}=\frac{7}{4}$

(b) $m_{AP}=\frac{2-1}{1-\frac{3}{4}}=4$; equation of PQ (PQ 的方程): $\frac{y-2}{x-1}=-\frac{1}{4}$, $x+4y-9=0$

(c) $7+4(k)-9=0$, $k=\frac{1}{2}$; $\therefore PQ=\sqrt{(7-1)^2+(\frac{1}{2}-2)^2}=\frac{\sqrt{153}}{2}=\frac{3\sqrt{17}}{2}$

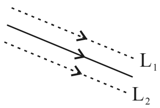
5. (a) $m_{OM}=\dfrac{2-0}{-1-0}=-2$

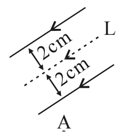
(b) $m_{PQ}=-1\div(-2)=\dfrac{1}{2}$, $\therefore\ \dfrac{y-2}{x+1}=\dfrac{1}{2}$, $x-2y+5=0$

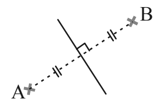
(c) (i) $r_2=OM=\sqrt{(-1-0)^2+(2-0)^2}=\sqrt{5}$

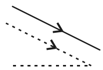
(ii) $x^2+y^2=5$

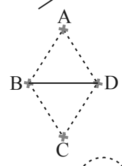
7-1

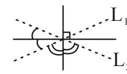
1. 

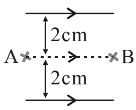
2. 

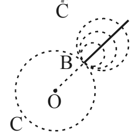
3. 

4. 

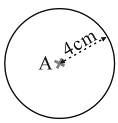
5. 

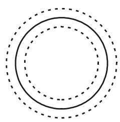
6. 

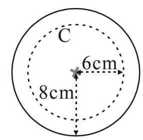
7. 

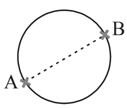
8. 

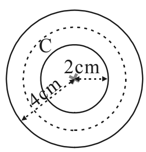
7-2

1. 

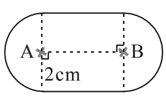
2. 

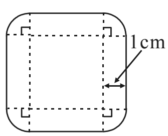
3. 

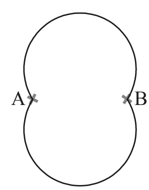
4. 

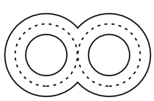
5. 

7-3

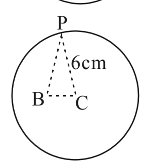
1. 

2. 

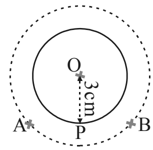
3. 

4. 

5. The locus is a circle with centre C and radius 6cm.
軌跡是一個圓形，圓心為 C，半徑為 6cm。



6. $OP=\sqrt{OA^2-AP^2}=\sqrt{5^2-4^2}=3$ cm
The locus is a circle with centre O and radius 3cm.
軌跡是一個圓形，圓心為 O，半徑為 3cm。



7-4

1. (a) $y\pm 3=0$ (b) $x\pm 6=0$ (c) $x+8=0$ or 或 $x-2=0$
(d) $2y+3=0$ or 或 $2y-5=0$ (e) $3x-7=0$ or 或 $3x-1=0$
2. (a) $x-1=0$ (b) $y+3=0$ (c) $x=\frac{13}{12}$, $12x-13=0$
(d) $y=-\frac{5}{8}$, $8y+5=0$ (e) $y=3x-5$ (f) $y=3x+\frac{3}{4}$, $12x-4y+3=0$
3. (a) $AB=3, h=4,\quad \therefore y-6=0$ or 或 $y+2=0$
(b) $AB=2, h=10,\quad \therefore x+6=0$ or 或 $x-14=0$
(c) $AB=7, h=2,\quad \therefore x+3=0$ or 或 $x-1=0$
(d) $AB=5, h=4.8,\quad \therefore 5y+39=0$ or 或 $5y-9=0$
4. (a) $y+5=0$ (b) $x+10=0$ (c) $x-2=0$
(d) $x-y=0$ or 或 $x+y=0$

7-5

1. (a) $(x+5)^2+(y-1)^2=(x-1)^2+(y-3)^2$, $3x+y+4=0$
(b) $(x+6)^2+(y-2)^2=(x+14)^2+(y-6)^2$, $2x-y+24=0$
(c) $(x-9)^2+(y+2)^2=(x-5)^2+(y+6)^2$, $x+y-3=0$
(d) $(x-15)^2+(y+8)^2=(x+7)^2+(y-1)^2$, $44x-18y-239=0$
(e) $(x+8)^2+(y+7)^2=(x-3)^2+(y-9)^2$, $22x+32y+23=0$
2. (a) $\sqrt{(x-9)^2+(y+8)^2}=2\sqrt{(x-9)^2+(y-4)^2}$
$(x-9)^2+(y+8)^2=4(x-9)^2+4(y-4)^2$, $x^2+y^2-18x-16y+81=0$
(b) $3\sqrt{(x+2)^2+(y-4)^2}=\sqrt{(x-1)^2+(y+3)^2}$, $4x^2+4y^2+19x-39y+85=0$
(c) $4\sqrt{(x-4)^2+(y-1)^2}=3\sqrt{(x+5)^2+(y+7)^2}$, $7x^2+7y^2-218x-158y-394=0$
(d) $2\sqrt{(x-7)^2+(y+2)^2}=5\sqrt{(x-1)^2+(y-3)^2}$, $21x^2+21y^2+6x-166y+38=0$
3. (a) $\sqrt{(x-4)^2+(y+2)^2}=4-y$, $x^2-8x+12y+4=0$
(b) $\sqrt{(x+9)^2+(y-5)^2}=y+1$, $x^2+18x-12y+105=0$
(c) $\sqrt{(x+2)^2+(y+3)^2}=3-x$, $10x+y^2+6y+4=0$
(d) $\sqrt{(x-5)^2+(y-5)^2}=x+6$, $22x-y^2+10y-14=0$
4. (a) $\sqrt{x^2+(y-1)^2}+\sqrt{(x-4)^2+(y-1)^2}=6$, $x^2+(y-1)^2=(6-\sqrt{(x-4)^2+(y-1)^2})^2$,
$3\sqrt{(x-4)^2+(y-1)^2}=13-2x$, $9(x^2-8x+16+y^2-2y+1)=169-42x+4x^2$,
$5x^2+9y^2-20x-18y-16=0$
(b) $\sqrt{(x-5)^2+y^2}+\sqrt{(x-2)^2+(y+1)^2}=4$, $(x-5)^2+y^2=(4-\sqrt{(x-2)^2+(y+1)^2})^2$
$4\sqrt{(x-2)^2+(y+1)^2}=3x+y-2$, $7x^2+15y^2-52x+36y-6xy+76=0$

7-6

1. (a) $(x+8)^2+(y+7)^2=5^2$, $x^2+y^2+16x+14y+88=0$

   (b) $(x+1)^2+(y-9)^2=7^2$, $x^2+y^2+2x-18y+33=0$

   (c) $(x-\frac{2}{3})^2+(y+\frac{17}{2})^2=(\frac{5}{6})^2$, $3x^2+3y^2-4x+51y+216=0$

2. (a) $\frac{y-7}{x+2}\cdot\frac{y+1}{x-4}=-1$, $x^2+y^2-2x-6y-15=0$

   (b) $\frac{y-1}{x-6}\cdot\frac{y-8}{x+2}=-1$, $x^2+y^2-4x-9y-4=0$

   (c) $\frac{y-\frac{3}{8}}{x-\frac{4}{5}}\cdot\frac{y+\frac{5}{6}}{x-\frac{13}{2}}=-1$, $240x^2+240y^2-1752x+110y+1173=0$

3. (a) centre 圓心: $(-2,-2)$, radius 半徑: 4

   $(x+2)^2+(y+2)^2=6^2$ or 或 $(x+2)^2+(y+2)^2=2^2$

   $x^2+y^2+4x+4y-28=0$ or 或 $x^2+y^2+4x+4y+4=0$

   (b) centre 圓心: $(-2,3)$, radius 半徑: 5

   $(x+2)^2+(y-3)^2=7^2$ or 或 $(x+2)^2+(y-3)^2=3^2$

   $x^2+y^2+4x-6y-36=0$ or 或 $x^2+y^2+4x-6y+4=0$

   (c) centre 圓心: $(4,-3)$, radius 半徑: $\frac{7}{2}$

   $(x-4)^2+(y+3)^2=(\frac{11}{2})^2$ or 或 $(x-4)^2+(y+3)^2=(\frac{3}{2})^2$

   $4x^2+4y^2-32x+24y-21=0$ or 或 $4x^2+4y^2-32x+24y+91=0$

   (d) centre 圓心: $(3,2)$, radius 半徑: $\frac{3}{2}$

   $(x-3)^2+(y-2)^2=(\frac{7}{2})^2$, $4x^2+4y^2-24x-16y+3=0$

4. (a) centre 圓心: $(3,4)$, radius 半徑: $r_1=2$, $r_2=3$

   $(x-3)^2+(y-4)^2=(\frac{3}{2})^2$, $4x^2+4y^2-24x-32y+91=0$

   (b) centre 圓心: $(-1,2)$, radius 半徑: $r_1=\frac{5}{2}$, $r_2=4$

   $(x+1)^2+(y-2)^2=(\frac{13}{4})^2$, $16x^2+16y^2+32x-64y-89=0$

   (c) centre 圓心: $(-3,-1)$, radius 半徑: $r_1=3$, $r_2=5$

   $(x+3)^2+(y+1)^2=4^2$, $x^2+y^2+6x+2y-6=0$

7-7

1. (a) $t=y+6$, $x-5y-37=0$ (b) $t=x-9$, $y=2x^2-36x+158$

   (c) $x^2=\cos^2 t$, $y^2=\sin^2 t$, $\therefore$ $x^2+y^2=1$

2. (a) Let 設 Q be 為 (a,b). $x=\frac{a-5}{2}$, $\therefore$ $a=2x+5$; $y=\frac{b+7}{2}$, $\therefore b=2y-7$;

   $2y-7=3(2x+5)+6$, $\therefore 3x-y+14=0$

(b) $a=2x-3$, $b=2y+4$; $2y+4=4(2x-3)^2-3(2x-3)+1$, $\therefore y=8x^2-27x+21$

(c) $a=2x+1$, $b=2y-2$;

$(2x+1)^2+(2y-2)^2-2(2x+1)-8=0$, $\therefore 4x^2+4y^2-8y-5=0$

(d) $a=2x-8$, $b=2y+9$; $2x-8=(2y+9)^2-2(2y+9)+7$, $\therefore x=2y^2+16y+39$

3. Let 設 Q be 為 $(a,0)$, R be 為 $(0,b)$.

(a) $x=\dfrac{1\cdot 0+2\cdot a}{1+2}$, $a=\dfrac{3x}{2}$; $y=\dfrac{1\cdot b+2\cdot 0}{1+2}$, $b=3y$;

$(\dfrac{3x}{2})^2+(3y)^2=7^2$, $\therefore 9x^2+36y^2-196=0$

(b) $a=\dfrac{5x}{3}$, $b=\dfrac{5y}{2}$; $(\dfrac{5x}{3})^2+(\dfrac{5y}{2})^2=12^2$, $\therefore 100x^2+225y^2-5184=0$

(c) $QP:PR=1:3$, $a=\dfrac{4x}{3}$, $b=4y$; $(\dfrac{4x}{3})^2+(4y)^2=9^2$, $\therefore 16x^2+144y^2-729=0$

(d) $QP:PR=5:2$, $a=\dfrac{7x}{2}$, $b=\dfrac{7y}{5}$;

$(\dfrac{7x}{2})^2+(\dfrac{7y}{5})^2=15^2$, $\therefore 1225x^2+196y^2-22500=0$

7-8

1. (a) slope of L (L的斜率) $=\dfrac{1}{4}$; $\therefore y-3=\dfrac{1}{4}(x+2)$, $x-4y+14=0$

(b) slope 斜率 $=-\dfrac{5}{2}$; $5x+2y-31=0$ (c) slope 斜率 $=2$; $2x-y-3=0$

2. (a) slope of locus 軌跡的斜率 $=1\div(-\dfrac{1}{2})=2$; $2x-y+3=0$

(b) slope of locus 軌跡的斜率 $=-1\div\dfrac{2}{3}=-\dfrac{3}{2}$; $3x+2y-26=0$

(c) slope of locus 軌跡的斜率 $=-1\div\dfrac{6}{7}=-\dfrac{7}{6}$; $7x+6y+34=0$

3. (a) $A=(4,6)$, $B=(8,0)$ (b) $2x-3y-3=0$

4. (a) $\sqrt{[x-(-1)]^2+(y-5)^2}=\sqrt{(x-3)^2+[y-(-5)]^2}$

$x^2+2x+1+y^2-10y+25=x^2-6x+9+y^2+10y+25$

$8x-20y-8=0$, $2x-5y-2=0$

(b) Put 把 $Q(-24,-10)$ into 代入 $2x-5y-2=0$,

L.H.S.左方 $=2(-24)-5(-10)-2=0=$ R.H.S.右方

$\therefore$ Q lies on the locus of P. Q 位於 P 的軌跡上。

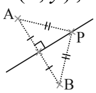
(c) L_1: $6x-15y+7=0$, $y=\dfrac{2}{5}x+\dfrac{7}{15}$, $m_{L_1}=\dfrac{2}{5}$

Locus of P (P 點軌跡的方程): $2x-5y-2=0$, $y=\dfrac{2}{5}x-\dfrac{2}{5}$, $m_P=\dfrac{2}{5}$, $\therefore$ Yes 是

(d) L_2: $4x+10y-3=0$, $y=-\dfrac{2}{5}x+\dfrac{3}{10}$, $m_{L_2}=-\dfrac{2}{5}$

$m_{L_2}\times m_P=-\dfrac{2}{5}\times\dfrac{2}{5}=-\dfrac{4}{25}\neq 1$, $\therefore$ No 不是

7-9

1. (a) A circle with centre A and radius 4. 以 A 為圓心及半徑為 4 的圓。
   (b) $P(x, y)$, $\sqrt{[x-(-6)]^2+(y-5)^2}=4$, $x^2+y^2+12x-10y+45=0$
2. (a) $P(x, y)$, $(x-2)^2+(y-3)^2=(x-7)^2+[y-(-7)]^2$, $2x-4y-17=0$
   (b) 

   It is the perpendicular bisector of line segment AB.
   它是線段 AB 的垂直平分線。
3. (a) A line // to L_1. 與 L_1 平行的直線。
   (b) slope 斜率 $=m_{L_1}=\frac{5}{4}$, y-intercept (y 軸截距) $=\frac{5+(-9)}{2}=-2$,
   Locus 軌跡: $y=\frac{5}{4}x-2$, $5x-4y-8=0$

8-1

1. (a) $m\times n$ (b) $m+n$ (c) $m\times n$ (d) $m+n$ (e) $m\times n$ (f) $m\times n$
2. (a) $4\times3=12$ (b) $5\times4=20$ (c) $4\times4=16$ (d) $5\times5=25$
3. (a) $6\times6\times5=180$ (b) $3\times3\times5+3\times4\times5=105$ (c) $2\times6\times5=60$

8-2

1. (a) 24 (b) 3628800 (c) 7 (d) 600 (e) 144
   (f) $=15\times14\times13=2730$
2. (a) 38! (b) 22! (c) $=\frac{40\times39\times...\times10\times...\times1}{9\times...\times1}=\frac{40!}{9!}$
   (d) $=\frac{55\times54\times...\times1}{16\times15\times...\times1}=\frac{55!}{16!}$
   (e) $=3(1)\times3(2)\times3(3)\times...\times3(33)=3^{33}(1\times2\times...\times33)=3^{33}\times33!$
   (f) $=2(30)\times2(29)\times...\times2(1)=2^{30}(30\times29\times...\times1)=2^{30}\times30!$
3. (a) $=\frac{6!}{(6-2)!}=6\times5=30$ (b) 12 (c) 1
   (d) $=\frac{4!}{(4-4)!}+\frac{5!}{(5-3)!}=4!+5\times4\times3=84$
4. (a) $=\frac{16!}{(16-10)!}=\frac{16!}{6!}$ (b) $=\frac{28!}{(28-3)!}=\frac{28!}{25!}$ (c) $=\frac{15!}{(15-15)!}=15!$
   (d) $=\frac{31!}{(31-9)!}=\frac{31!}{22!}$

8-3

1. $P_6^6=6!=720$ 2. $P_4^{12}=11880$ 3. $P_3^{23}=10626$
4. (a) $P_2^8=56$ (b) $P_5^8=6720$ (c) $P_8^8=8!=40320$
5. (a) $P_6^{16}=5765760$ (b) $P_6^7=5040$ (c) $P_6^9=60480$
6. (a) $P_2^4\times P_5^5=1440$ (b) $6\times P_6^6=4320$ (c) $3\times P_6^6=2160$
   (d) $(4+1)!\times3!=720$

7. (a) $8! = 40320$ (b) $2 \times 4! \times 4! = 1152$
(c) possible arrangements 可能的排列: BGBGBGBG, GBGBGBGB. $\therefore 2 \times 4! \times 4! = 1152$

8-4

1. $4 \times P_2^4 = 48$
2. $5 \times P_3^5 = 300$
3. $3 \times P_2^4 = 36$
4. $2 \times 4 \times P_2^4 + P_3^5 = 156$
5. $5 \times P_2^5 + P_3^6 = 220$
6. $3 \times P_3^5 + 5 \times P_5^5 = 1380$
7. $1 \times 2 \times 3 + 2 \times P_2^4 + P_2^5 + 5 = 55$

8-5

1. (a) $P_8^8 - P_7^7 \times P_2^2 = 30240$ (b) $P_8^8 - P_6^6 \times P_3^3 = 36000$
2. (a) $P_8^8 = 40320$ (b) $P_5^5 \times P_4^4 = 2880$ (c) $P_8^8 - C_4^5 \times P_4^4 \times P_4^4 = 37440$
3. $P_6^6 - P_3^3 \times P_3^3 \times 2 = 648$
4. $P_4^7 - P_5^5 = 720$
5. (a) $1 \times P_6^6 = 720$ (b) $1 \times P_6^6 - 1 \times 1 \times P_5^5 = 600$

8-6

1. (a) 16 (b) 1
(c) $\frac{7 \times 6}{2} = 21$ (d) $= \frac{6 \times 5 \times 4}{1 \times 2 \times 3} = 20$ (e) $= C_2^{20} = \frac{20 \times 19}{2} = 190$
2. (a) $= \frac{8!}{(8-5)!\,5!} = \frac{8!}{3!\,5!}$ (b) $= \frac{11!}{(11-6)!\,6!} = \frac{11!}{5!\,6!}$ (c) $= \frac{23!}{(23-7)!\,7!} = \frac{23!}{16!\,7!}$
(d) $= \frac{32!}{(32-19)!\,19!} = \frac{32!}{13!\,19!}$
3. (a) n (b) $\frac{n(n-1)}{2}$ (c) $= \frac{n(n-1)(n-2)}{1 \times 2 \times 3} = \frac{n(n-1)(n-2)}{6}$
(d) $= C_2^n = \frac{n(n-1)}{2}$
4. (a) 141 (b) 192 (c) 20 (d) $\frac{91}{45}$
5. (a) $\frac{n(n-1)}{2} = 36$, $n^2 - n - 72 = 0$, $n = 9$ or -8 (rejected 捨去)
(b) $\frac{(n+1)n}{2} = 105$, $n^2 + n - 210 = 0$, $n = 14$ or -15 (rejected 捨去)
(c) $C_{10-k}^{10} = C_{k+2}^{10}$, $\therefore 10 - k = k + 2$, $k = 4$

8-7

1. (a) $C_2^8 = 28$ (b) $C_3^8 = 56$
2. $C_4^6 \times C_3^5 \times C_2^3 = 450$
3. $C_5^{12} - (C_7^8 + C_5^8) = 728$ (OR 或: $C_4^8 \times C_1^4 + C_3^8 \times C_2^4 + C_2^8 \times C_3^4 = 728$)
4. (a) $C_5^{13} - C_4^6 \times C_1^7 - C_5^6 = 1176$ (OR 或: $C_0^6 \times C_5^7 + C_1^6 \times C_4^7 + C_2^6 \times C_3^7 + C_3^6 \times C_2^7 = 1176$)
(b) $C_5^{13} - C_4^4 \times C_1^9 = 1278$ (OR 或: $C_0^4 \times C_5^9 + C_1^4 \times C_4^9 + C_2^4 \times C_3^9 + C_3^4 \times C_2^9 = 1278$)

5. (a) $C_4^8 \times C_2^9 = 2520$ (b) $2520 + C_5^8 \times C_1^9 + C_6^8 \times C_0^9 = 3052$

6. $C_3^6 - C_2^2 \times C_1^4 = 16$ (OR 或: $C_3^4 + C_2^4 + C_2^4 = 16$)

7. (a) $4 \times C_5^{13} = 5148$ (b) $13 \times C_1^{48} = 624$ (c) $13 \times C_3^4 \times 12 \times C_2^4 = 3744$

8-8

1. P_6^{10}, $C_6^{10} \times 6!$
2. P_3^{10}, $C_3^{10} \times 3!$
3. C_6^{49}
4. P_4^6, $C_4^6 \times 4!$
5. P_6^{10}, $C_6^{10} \times 6!$
6. $C_2^5 \times 1$
7. $C_4^6 \times C_3^7 \times 7!$
8. $C_8^{15} \times 6! \times 2$
9. $C_4^{26} \times 2$
10. $6!$, P_6^6

8-9

1. (a) (i) $P_2^4 = 12$ (ii) $4^2 = 16$ (b) (i) $P_3^5 = 60$ (ii) $5^3 = 125$
   (c) (i) $5 \times P_3^5 = 300$ (ii) $5 \times 6^3 = 1080$
2. (a) $C_2^5 = 10$ $(\frac{5!}{3!\,2!})$ (b) $C_3^8 = 56$ $(\frac{8!}{5!\,5!})$ (c) $C_4^8 = 70$ $(\frac{8!}{4!\,4!})$
3. (a) $C_3^6 = 20$ $(\frac{6!}{3!\,3!})$ (b) $C_4^9 = 126$ $(\frac{9!}{4!\,5!})$ (c) 0 (d) $C_6^9 = 84$ $(\frac{9!}{6!\,3!})$

8-10

1.

From P (由 P)	Possible larger numbers from Q (由 Q 得到的大數)
0	1, 2, 4, 7, 9
3	4, 7, 9
4	7, 9
7	9

$\therefore$ the total no. of ways 共有的方法 $= 5 + 3 + 2 + 1 = 11$

2. (a) $P_3^{10} = 720$ (b) $10^3 = 1000$ 3. $2^5 = 32$ 4. $P_3^4 = 24$

5. $\because$ No. of line segments from joining any 2 points. 任意連結兩點所得的線段數目 $= C_2^n$,
   but n of the above line segments are sides of the polygon 當中有 n 條是多邊形的邊,
   $\therefore$ the total no. of diagonals 對角線的數目 $= C_2^n - n = \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$

6. $13 \times (C_2^{13} \times C_3^{26} + C_3^{13} \times C_2^{26} + C_4^{13} \times C_1^{26} + C_5^{13}) = 4103151$
   [OR 或: $13 \times (C_5^{39} - C_5^{26} - C_1^{13} \times C_4^{26} = 4103151$]

7. $P_{11}^{11} = 39{,}916{,}800$

8. (a) From the 1st to the 8th ball, any 4 can be the red balls.
   由第 1 個到第 8 個，其中任何 4 個都可以是紅球。 $\therefore$ $C_4^8 = 70$
   (b) Treating the 4 yellow balls as one single object, there will be 5 possible positions for them. 把 4 個黃球當作一個整體，共有 5 個位置可以擺放它們。
   1 (Red 紅) 2 (Red 紅) 3 (Red 紅) 4 (Red 紅) 5
   $\therefore$ No. of possible arrangements 排列方法 $= 5$
   (c) There are only 2 possible ways. 只有 2 個排列方法。 (RYRYRYRY, YRYRYRYR)

9. (a) $P_8^8 \times P_2^2 = 80640$

(b) To separate one row into 2 rows, we can have the first 4 students in the first row, and the 5^{th} to 9^{th} students in the second row. Then from the arrangements in Part (a), we exclude those cases in which the two particular students are at the 4^{th} and the 5^{th} places.
把一行分為 2 行，我們可以把首 4 個學生放在第一行, 第 5 至第 9 個學生放在第二行。利用(a)部份的排列方法，然後排除兩名指定學生站在第 4 及第 5 個位置的情況。

$\therefore$ No. of possible arrangements 排列方法 $= P_8^8 \times P_2^2 - P_7^7 \times P_2^2 = 70560$

8-11

1. $P_3^4 \times P_4^5 = 2880$
2. (a) $P_2^9 = 72$ (b) $P_2^9 - P_2^5 = 52$ ($\because$ 5 odd digits 五個單數數字)
3. $C_4^{24} - C_4^{15} = 9261$
4. (a) $C_4^{16} = 1820$ (b) $C_3^{10} \times C_1^6 = 720$ (c) $C_4^{16} - C_4^6 - C_1^{10} \times C_3^6 = 1605$
5. (a) $C_4^{24} = 10626$

   (b) (i) $C_4^{12} = 495$ (ii) $C_4^{24} - C_4^{12} - C_3^{12} \times C_1^{12} = 7491$

   (iii) $C_2^{12} = 66$ (iv) $C_4^{12} \times 2^4 = 7920$ (v) $C_4^{24} - C_4^{12} \times 2^4 = 2706$

9-1

1. $\dfrac{4}{11}$ 2. $\dfrac{24-11-7}{24} = \dfrac{1}{4}$ 3. $\dfrac{8}{20} = \dfrac{2}{5}$ 4. $\dfrac{30}{365} = \dfrac{6}{73}$
5. (a) $\dfrac{3\times4}{52} = \dfrac{3}{13}$ (b) $\dfrac{3}{52}$ (c) $\dfrac{52-13-3}{52} = \dfrac{9}{13}$ 6. $\dfrac{56+64+40}{200} = \dfrac{4}{5}$
7. $25000 \times 0.0012 = 30$
8. $\dfrac{x}{14+16+x} = \dfrac{2}{3}$, $x = 60$, the total number 總數 $= 60+14+16 = 90$

9-2

1. (a) $\dfrac{9}{36} = \dfrac{1}{4}$ (b) $\dfrac{36-6}{36} = \dfrac{5}{6}$ (c) $\dfrac{15}{36} = \dfrac{5}{12}$ (d) $\dfrac{2+5+4+1}{36} = \dfrac{1}{3}$
2. (a) $\dfrac{3}{8}$ (b) $\dfrac{4}{8} = \dfrac{1}{2}$ (c) $\dfrac{7}{8}$
3. (a) 25 (b) $\dfrac{12}{25}$
4. (a)

	M	A	G	G	I	E
M	╳	MA	MG	MG	MI	ME
A	AM	╳	AG	AG	AI	AE
G	GM	GA	╳	GG	GI	GE
G	GM	GA	GG	╳	GI	GE
I	IM	IA	IG	IG	╳	IE
E	EM	EA	EG	EG	EI	╳

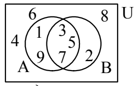
(b) (i) $\dfrac{2}{30} = \dfrac{1}{15}$

(ii) $\dfrac{6}{30} = \dfrac{1}{5}$

(iii) $\dfrac{18}{30} = \dfrac{3}{5}$

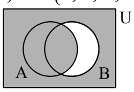
9-3

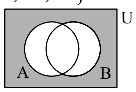
1. $\{a, b, e, f\}$

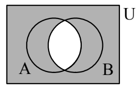
2. (a) 

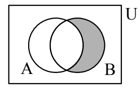
2. (b) (i) $\{3, 5, 7\}$ (ii) $\{1, 2, 3, 5, 7, 9\}$ (iii) $\{2, 3, 4, 5, 6, 7, 8\}$ (iv) $\{1, 9\}$

3. (a) (i) $\{e, l, \text{m}, \text{n}, \text{t}\}$ (ii) 5
   (b) (i) $\{2, 3, 5, 7, 11\}$ (ii) 5
   (c) (i) $\{0, 1, 2\}$ (ii) 3
   (d) (i) $\{-1, 1\}$ (ii) 2
   (e) (i) $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ (ii) 11

4. (a) 

(b) 

(c) 

(d) 

5. (a) $(9+3-2)+8=18$
   (b) $60-(23+28-8)=17$ (c) $16+33+5-42=12$

9-4

1. (a) A and 和 C, B and 和 C (b) A and 和 B, B and 和 C
   (c) A and 和 B

2. 0.63 3. 0.92 4. $1-P(\text{FFF 女女女})=1-\frac{1}{8}=\frac{7}{8}$

5. $1-P(\text{TTT 反反反})=1-\frac{1}{8}=\frac{7}{8}$

6. $1-P(\text{both numbers are odd 兩個奇數點數})=1-\frac{9}{36}=\frac{3}{4}$

9-5

1. (a) $0.081+0.265=0.346$ (b) $1-0.346=0.654$

2. (a) $\frac{4}{52}=\frac{1}{13}$ (b) $\frac{26}{52}=\frac{1}{2}$ (c) $\frac{1}{13}+\frac{1}{2}-\frac{2}{52}=\frac{7}{13}$

3. (a) (i) $\frac{10}{36}=\frac{5}{18}$ (ii) $\frac{6}{36}=\frac{1}{6}$ (iii) $\frac{5}{18}+\frac{1}{6}=\frac{4}{9}$
   (b) (i) $\frac{6}{36}=\frac{1}{6}$ (ii) $\frac{6}{36}=\frac{1}{6}$ (iii) $\frac{1}{6}+\frac{1}{6}-\frac{1}{36}=\frac{11}{36}$
   (c) (i) $\frac{5}{36}$ (ii) $\frac{9}{36}=\frac{1}{4}$ (iii) $\frac{5}{36}+\frac{1}{4}-\frac{3}{36}=\frac{11}{36}$

4. (a) (i) $0.38+0.45-0.73=0.1$ (ii) No 否
   (b) (i) $0.52+0.29-0.81=0$ (ii) Yes 是
   (c) (i) $0.24+0.77-0.16=0.85$ (ii) No 否

5. (a) $0.3+0.5-0.12=0.68$ (b) $1-0.68=0.32$

9-6

1. (a) Yes 是 (b) Yes 是 (c) No 否 (d) Yes 是

2. (a) $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$ (b) $1-P(\text{HHH 正正正})=1-\frac{1}{8}=\frac{7}{8}$

3. (a) $\frac{3}{6}\times\frac{3}{6}=\frac{1}{4}$ (b) $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$ (c) $1-\frac{5}{36}\times\frac{5}{36}=\frac{11}{36}$

4. (a) $\frac{8}{14}\times\frac{8}{14}=\frac{16}{49}$ (b) $\frac{6}{14}\times\frac{6}{14}=\frac{9}{49}$ (c) $1-\frac{16}{49}=\frac{33}{49}$

9-7

1. (a) $\frac{6}{4+6}\times\frac{5}{4+5}=\frac{1}{3}$ (b) $\frac{4}{4+6}\times\frac{6}{3+6}+\frac{6}{4+6}\times\frac{4}{4+5}=\frac{8}{15}$

2. (a) $\frac{12}{52}\times\frac{11}{51}=\frac{11}{221}$ (b) $\frac{12}{52}\times\frac{40}{51}+\frac{40}{52}\times\frac{12}{51}=\frac{80}{221}$

9-8

1. (a) $\frac{6}{10}\times\frac{4}{10}+\frac{4}{10}\times\frac{6}{10}=\frac{12}{25}$ (b) $\frac{6}{10}\times\frac{4}{9}+\frac{4}{10}\times\frac{6}{9}=\frac{8}{15}$

2. (a) $\frac{26}{52}\times\frac{25}{51}+\frac{26}{52}\times\frac{25}{51}=\frac{25}{51}$ (b) $\frac{13}{52}\times\frac{13}{51}+\frac{13}{52}\times\frac{13}{51}=\frac{13}{102}$

3. (a) $\frac{18}{40}\times\frac{24}{32}=\frac{27}{80}$ (b) $\frac{18}{40}\times\frac{8}{32}+\frac{22}{40}\times\frac{24}{32}=\frac{21}{40}$
   (c) $\frac{27}{80}+\frac{21}{40}=\frac{69}{80}$

4. (a) $0.84\times0.36=0.3024$ (b) $0.84\times(1-0.36)+(1-0.84)\times0.36=0.5952$
   (c) $0.3024+0.5952=0.8976$

5. $0.4\times\frac{2}{6}+0.37\times\frac{2}{5}+0.23\times\frac{4}{5}=0.4653$

6. (a) $(1-0.25)\times0.72\times(1-0.6)=0.216$
   (b) $0.25\times(1-0.72)\times(1-0.6)+0.216+(1-0.25)\times(1-0.72)\times0.6=0.37$
   (c) $0.25\times0.72\times(1-0.6)+0.25\times(1-0.72)\times0.6+(1-0.25)\times0.72\times0.6=0.438$
   (d) $1-(1-0.25)(1-0.72)(1-0.6)=0.916$

7. (a) $\frac{1}{6}\times\frac{3}{4}=\frac{1}{8}$ (b) $1-\frac{1}{8}=\frac{7}{8}$

9-9

1. (a)

	no. of girls 女生數目	no. of boys 男生數目
like ice-cream 愛吃雪糕	18	4
not like ice-cream 不愛吃雪糕	6	12

   (b) $\frac{18}{22}=\frac{9}{11}$
   (c) $\frac{12}{16}=\frac{3}{4}$

2. (a) $\frac{2}{5}$ (b) $\frac{2}{5}$ (c) $\frac{2}{4}=\frac{1}{2}$ (d) $\frac{2}{5}$ (e) $\frac{3}{5}$ (f) $\frac{3}{5}$

3. (a) $\frac{7}{7+12}=\frac{7}{19}$ (b) $\frac{8}{8+11}=\frac{8}{19}$ (c) $\frac{12}{7+12}=\frac{12}{19}$ (d) $\frac{11}{8+11}=\frac{11}{19}$

4. (a) 0 (b) $\frac{1}{1+2+2}=\frac{1}{5}$ (c) $\frac{2}{1+2+2}=\frac{2}{5}$

(d) $\frac{3}{3+2}=\frac{3}{5}$ (e) $\frac{2}{1+2+2}=\frac{2}{5}$

9-10

1. (a) 0.6163 (b) 0.1628 (c) 0.1994 (d) 0.1598
2. $P(\text{F.1中一} \cap \text{Boy男生}) = P(\text{F.1中一}) \times P(\text{Boy男生} \mid \text{F.1中一}) = 0.284 \times 0.653 = 0.1855$
3. $\frac{0.4}{0.75}=\frac{8}{15}$ 4. $\frac{0.7}{0.84}=\frac{5}{6}$ 5. $\frac{0.0084}{0.0126}=\frac{2}{3}$ 6. $\frac{0.185}{0.45}=\frac{37}{90}$

9-11

1. (a) $\frac{1\times P_4^4+1}{P_5^5}=\frac{5}{24}$ (b) $\frac{2\times P_3^3}{P_5^5}=\frac{1}{10}$ (c) $\frac{P_4^4\times P_5^5}{P_8^8}=\frac{1}{14}$
2. (a) $\frac{1\times 4\times P_4^4}{P_6^6}=\frac{2}{15}$ (b) $\frac{3\times 3\times P_4^4+2\times 4\times P_4^4}{P_6^6}=\frac{17}{30}$
3. (a) $\frac{C_3^8\times C_1^5+C_4^8}{C_4^{13}}=\frac{70}{143}$ (b) $\frac{C_3^{11}}{C_5^{13}}=\frac{5}{39}$ (c) $\frac{C_3^4\times C_2^7}{C_4^5\times C_3^8}=\frac{3}{10}$

9-12

1. (a) $(\frac{1}{2})^3(\frac{1}{2})^2C_3^5=\frac{5}{16}$ (b) $1-(\frac{1}{2})^5-(\frac{1}{2})(\frac{1}{2})^4C_1^5=\frac{13}{16}$
2. (a) $(\frac{3}{6})(\frac{3}{6})^5C_1^6=\frac{3}{32}$ (b) $(\frac{1}{6})^3(\frac{5}{6})^3C_3^6=0.0536$ (c) $(\frac{3}{6})^5(\frac{3}{6})C_5^6+(\frac{3}{6})^6=\frac{7}{64}$
3. (a) $(\frac{1}{2})(\frac{1}{2})^3C_1^4=\frac{1}{4}$ (b) $(\frac{1}{2})^2(\frac{1}{2})^2C_2^4=\frac{3}{8}$ (c) $(\frac{1}{2})^3(\frac{1}{2})C_3^4+(\frac{1}{2})^4=\frac{5}{16}$

9-13

1. (a) $\frac{2}{8}\times\frac{1}{9}+\frac{2}{8}\times\frac{1}{9}=\frac{1}{18}$ (b) $1-\frac{4}{8}\times\frac{7}{9}=\frac{11}{18}$
2. (a) $\frac{12}{49}$ (b) $\frac{7}{49}=\frac{1}{7}$ (c) $\frac{12}{49}+\frac{1}{7}-\frac{1}{49}=\frac{18}{49}$ (d) $\frac{\frac{1}{49}}{\frac{1}{7}}=\frac{1}{7}$
3. After the 2 balls are drawn, 7 red balls and 4 green balls are left.
抽出該兩個球後，剩餘 7 個紅球和 4 個綠球。
$\therefore$ The required probability 所求的概率 $=\frac{7}{11}$
4. $\frac{2^4}{9\times 10^3}=\frac{2}{1125}$ (or 或: $\frac{2}{9}\times\frac{2}{10}\times\frac{2}{10}\times\frac{2}{10}$)
5. $\frac{C_2^{13}\times C_3^{26}}{C_5^{52}}=0.0780$ 6. (a) $16\times\frac{1}{4}=4$ (b) $(\frac{1}{4})^4(\frac{3}{4})^{12}C_4^{16}=0.2252$

7. (a) $\frac{5}{6}\times\frac{1}{5}=\frac{1}{6}$ (b) $1-\frac{1}{6}-\frac{1}{6}=\frac{2}{3}$ (c) $\frac{3}{4}\times\frac{2}{3}\times\frac{1}{2}=\frac{1}{4}$

8. $\frac{5}{12}\times\frac{4+3}{11}+\frac{4}{12}\times\frac{3}{11}+\frac{3}{12}\times\frac{5+2}{11}=\frac{17}{33}$

9. $\frac{C_2^6+C_2^4+C_2^5}{C_2^5}=0.2952$

9-14

1. $(\frac{1}{52})^2(\frac{51}{52})\times C_2^3=0.0011$

2. Given that each of the four outcomes must be "5" or "6". The two "5"s can appear in any of the four outcomes.
已知擲四次而每次的結果都是那兩個"5"或"6"，那兩個"5"可出現於四次之中任何兩次。
$\frac{C_2^4}{2^4}=\frac{3}{8}$ {or 或: $[(\frac{1}{6})^2(\frac{1}{6})^2C_2^4]\div[\frac{2}{6}]^4=\frac{1}{216}\div\frac{1}{81}=\frac{3}{8}$}

3. (a) $\frac{5}{8}\times\frac{1}{9}=\frac{5}{72}$ (b) $\frac{3}{8}\times\frac{3}{9}+\frac{5}{8}\times\frac{2}{9}=\frac{19}{72}$

(c) $\frac{P(\text{the ball is yellow and from bag P 來自P袋的黃球})}{P(\text{the ball is yellow黃球})}=\frac{\frac{3}{8}\times\frac{1}{9}}{\frac{19}{72}}=\frac{3}{19}$

4. (a) $1-P(\text{all wrong 答錯所有問題})=1-(1-0.45)(1-0.6)(1-0.75)=0.945$

(b) (i) $[1-(1-0.45)^2]\times[1-(1-0.6)^2]\times[1-(1-0.75)^2]=0.5493$

(ii) $1-(1-0.45^2)(1-0.6^2)(1-0.75^2)=0.7767$

5. (a) Assume he selects a certain lock first. 假設他先選了某一個鎖。
$\therefore P(\text{the key fits the lock 鑰匙能夠解鎖})=\frac{1}{5}$

(b) $\frac{C_2^3}{C_2^5}=\frac{3}{10}$

6. No. of students taking 選修該科的學生人數:
Chem 化學 $=200\times45\%=90$, Phy 物理 $=200\times60\%=120$,
Chem or Phy 化學或物理 $=200\times72\%=144$
Chem and Phy 化學及物理 $=90+120-144=66$
Chem but not Phy 化學但沒選修物理 $=90-66=24$
neither Chem nor Phy 既沒修化學亦沒修物理 $=200-144=56$

(a) (i) $=\frac{66}{200}=0.33$ (ii) $=\frac{24}{200}=0.12$ (iii) $=\frac{56}{200}=0.28$

(b) $=\frac{\text{No. of students take Chem and Phy 選修化學及物理人數}}{\text{No. of students take Chem 選修化學人數}}=\frac{66}{90}=\frac{11}{15}$

(c) $=1-P(\text{both do not take Phy 兩名學生都沒有選修物理})$
$=1-\frac{80}{200}\times\frac{79}{199}=0.8412$

Alternative method 其他方法 [6(a), 6(b)]:

(a) (i) C: take Chemistry 選修化學, P: take Physics 選修物理;

$P(\text{C}) = 0.45$, $P(\text{P}) = 0.6$, $P(\text{C} \cup \text{P}) = 0.72$

$P(\text{C} \cap \text{P}) = 0.45 + 0.6 - 0.72 = 0.33$

(ii) $P(\text{C} \cap \text{P}') = P(\text{C}) - P(\text{C} \cap \text{P}) = 0.45 - 0.33 = 0.12$

(iii) $P(\text{C}' \cap \text{P}') = 1 - P(\text{C} \cup \text{P}) = 1 - 0.72 = 0.28$

(b) $P(\text{P} \mid \text{C}) = \dfrac{P(\text{P} \cap \text{C})}{\text{P(C)}} = \dfrac{0.33}{0.45} = \dfrac{11}{15}$

9-15

1. $1 - 0.37 - 0.15 - 0.06 = 0.42$

2. even 偶數: 2, 4, 6. $\therefore \dfrac{2}{3}$

3. = P(first digit is 1 首個數位是 1) = $\dfrac{1}{4}$

4. $1 - x - y$

5. $(\dfrac{1}{6})(\dfrac{5}{6})^2 \times 3 = \dfrac{25}{72}$

6. $(\dfrac{5}{6})^3(\dfrac{1}{6}) = \dfrac{125}{1296}$

7. $\dfrac{1 \times C_1^3}{C_2^5} = \dfrac{3}{10}$

8. $\dfrac{8}{12} \times \dfrac{4}{11} = \dfrac{16}{33}$ (Or 或: $\dfrac{C_1^8 \times C_1^4}{C_2^{12}} \times 2$)

9. blue 藍: $10 - 2 - 5 = 3$, $\dfrac{3}{10} \times \dfrac{7}{9} = \dfrac{7}{15}$ (Or 或: $\dfrac{C_1^3 \times C_1^7}{C_2^{10}} \times 2$)

10. total 共 $6 + 5 + 2 = 13$ coins 硬幣,

$\dfrac{6}{13} \times \dfrac{5}{12} + \dfrac{5}{13} \times \dfrac{4}{12} + \dfrac{2}{13} \times \dfrac{1}{12} = \dfrac{1}{3}$ (Or 或: $\dfrac{C_2^6 + C_2^5 + C_2^2}{C_2^{13}}$)

11. = 1 – P(none of them can shoot the bird 沒有人擊中小鳥) $= 1 - \dfrac{2}{5} \times \dfrac{5}{6} = \dfrac{2}{3}$

12. = 1 – P(all incorrect 全部答錯) $= 1 - (\dfrac{3}{4})^{10} = 0.944$

13. 1 – P(fail in both subjects 兩科均不合格) $= 0.92$, $1 - (1 - k)(1 - 0.68) = 0.92$, $k = 0.75$

14. P(at least one price slip 最少一張贈券) = 1 – P (no price slip 沒有贈券),

$1 - (\dfrac{14}{15})^n > 0.95$, $\log 0.05 > n \log(\dfrac{14}{15})$, $n > 43.4$, $\therefore 44$

9-16

1. $0.6 \times (1 - 0.25) + 0.4 = 0.85$

2. $\$2 + \$5 = \$7$, $\dfrac{3}{12} \times \dfrac{9}{11} = \dfrac{9}{22}$ (Or 或: $\dfrac{C_1^3 \times C_1^9}{C_2^{12}} \times 2$)

3. W: white 白, B: black 黑,

(a) 3W2B, 4W1B, $\therefore \dfrac{C_3^4 C_2^{10} + C_4^4 C_1^{10}}{C_5^{14}} = \dfrac{95}{1001}$

(b) Not 不是 1B4W, $\therefore 1 - \dfrac{C_1^{10} C_4^4}{C_5^{14}} = \dfrac{996}{1001}$

4. (a) $\frac{10\times2}{10\times10}=\frac{1}{5}$ [= P(last digit is 0 or 5 最尾數位是 0 或 5) $=\frac{2}{10}=\frac{1}{5}$]

(b) Divisible by 7 (能被 7 整除): 700, 707, 714, … , 798;

Total 15 numbers (共 15 個), $\therefore \frac{15}{10\times10}=\frac{3}{20}$

5. (a) $\frac{9\times9\times9}{900}=\frac{81}{100}$ (b) $\frac{9\times1\times1}{900}=\frac{1}{100}$ (c) $1-\frac{81}{100}=\frac{19}{100}$

6. $(\frac{6}{10})^3(\frac{4}{10})=\frac{54}{625}$

7. (a) = 1 – P(shortest boys are not chosen 沒有選出最矮的男生)

$=1-\frac{8}{10}\times\frac{7}{9}=\frac{17}{45}$ (Or 或: $1-\frac{C_2^8}{C_2^{10}}$)

(b) $\frac{2}{10}\times\frac{8}{9}\times2=\frac{16}{45}$ (Or 或: $\frac{C_1^2\times C_1^8}{C_2^{10}}$)

(c) = 1 – P(both shortest boys are chosen 選出最矮的兩名男生)

$=1-\frac{2}{10}\times\frac{1}{9}=\frac{44}{45}$ (Or 或: $1-\frac{1}{C_2^{10}}$) (Or 或: $\frac{8}{10}\times\frac{7}{9}+\frac{16}{45}$)

8. (a) (i) $\frac{13+21+18}{120}=\frac{13}{30}$ (ii) $\frac{54+21}{120}=\frac{5}{8}$

(iii) Level 5 (5 級): $6+13=19$, $\therefore \frac{13}{19}$

(b) (i) $\frac{54}{120}\times\frac{53}{119}=\frac{477}{2380}$ (Or 或: $\frac{C_2^{54}}{C_2^{120}}$)

(ii) $\frac{19}{120}\times\frac{18}{119}+\frac{32}{120}\times\frac{31}{119}+\frac{35}{120}\times\frac{34}{119}+\frac{34}{120}\times\frac{33}{119}=\frac{1823}{7140}$

(Or 或: $\frac{C_2^{19}+C_2^{32}+C_2^{35}+C_2^{34}}{C_2^{120}}$)

10-1

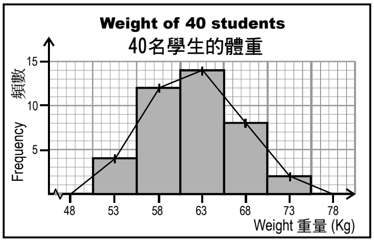
1.

Class boundaries 組界 (cm)	Class mark 組中點 (cm)
119.5 – 124.5	122
124.5 – 129.5	127
129.5 – 134.5	132
134.5 – 139.5	137

2. (a) 51kg, 55kg

(b) 70.5 - 75.5kg

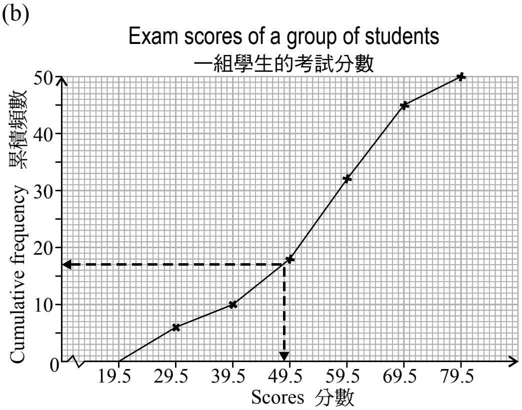
(c)



3. (a)

Score less than 分數少於	Cumulative frequency 累積頻數
19.5	0
29.5	6
39.5	10
49.5	18
59.5	32
69.5	45
79.5	50

(b)



(c) 17 students have scores less than 47.5 (17 名學生的分數少於 47.5),

$\therefore$ the pass percentage 合格率 $= \dfrac{50-17}{50} \times 100\% = 66\%$

10-2

1. (a) mean 平均數 $= 5°\text{C}$, mode 眾數 $= 14°\text{C}$, median 中位數 $= 6°\text{C}$

(b) mean 平均數 $= 20.5$, no mode 沒有眾數, median 中位數$= \dfrac{19.7+20.7}{2} = 20.2$

(c) 8 mm, 10 mm, 15 mm, 15 mm, 17 mm, 19 mm, 20 mm, 20 mm
mean 平均數 $= 15.5\text{mm}$, mode 眾數 $= 15\text{mm}, 20\text{mm}$,
median 中位數 $= \dfrac{15+17}{2} = 16\text{ mm}$

2. mean 平均數 $= \$37.2$, median 中位數 $= \dfrac{35+42}{2} = \$38.5$, mode 眾數 $= \$42$

3. (a) mean 平均數 $= \dfrac{37\times6+42\times17+47\times13+52\times8+57\times6}{6+17+13+8+6} = 46.1\text{kg}$

(b) 40kg – 44kg

4. When c.f. (累積頻數) = 24, salary (薪金) = \$10,900, $\therefore$ median 中位數 = \$10,900

5. $\dfrac{60\times12+50\times13}{25} = 54.8\text{ kg}$ 6. $16\times8-15\times7 = 23$

7. n = number of students 學生人數, $\dfrac{64n-48+84}{n} = 66$, $n = 18$

8. (a) 18 (b) 9, 10, 11

10-3

1.

	range 分佈域	median 中位數	Q_1	Q_3	Interquartile range 四分位數間距
(a)	21g	13.5g	10g	22g	12g
(b)	40	69	53.5	76	22.5
(c)	29cm	164.5cm	160cm	177cm	17cm

2. (a) Group A 組: $Q_1 = 314$, $Q_3 = 339.5$, interquartile range 四分位數間距 $= 25.5$
Group B 組: $Q_1 = 315$, $Q_3 = 337$, interquartile range 四分位數間距 $= 22$
(b) Group A 組. Its interquartile range is greater. 它的四分位數間距較大。

3. (a)

	Group A 組	Group B 組
range 分佈域	$= 41 - 20 = 21$kg	$= 45 - 24 = 21$kg
interquartile range 四分位數間距	$= 35.5 - 28 = 7.5$kg	$= 42 - 28 = 14$kg

(b) Group B 組

10-4

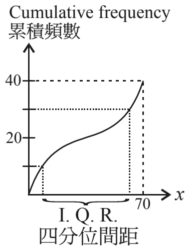
1. A: range 分佈域$= 94 - 47 = 47$, median 中位數 $= 73$, $Q_1 = 62$, $Q_3 = 82$,
interquartile range 四分位數間距 $= 20$
B: range 分佈域$= 79 - 42 = 37$, median 中位數 $= 65$, $Q_1 = 63$, $Q_3 = 74$,
interquartile range 四分位數間距 $= 11$
2. (a) David 大偉: max.最大值: 10h, min.最小值: 1h
Peter 彼得: max.最大值: 10h, min.最小值: 5h
(b) David 大偉: $6 - 3 = 3$h; Peter 彼得: $8 - 6 = 2$h
(c) David. The range and the interquartile range of his data are greater.
大偉。他的數據的分佈域及四分位數間距較大。
(d) Peter 彼得

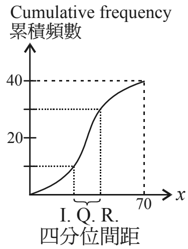
10-5

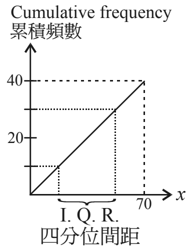
1. (a) mean 平均數 $= 30.6$; $\sigma = 7.76$ (b) mean 平均數$= 0$; $\sigma = 3.16$
(c) mean 平均數 $= 154.3$cm; $\sigma = 5.79$ cm
2. (a) Set A（甲組） (b) $\sigma_A = 1.83$, $\sigma_B = 1.17$. Yes 是, $\because \sigma_A > \sigma_B$.
3. (a) Group A（甲組）: mean 平均數 $= 48.9$kg; $\sigma = 2.75$ kg
Group B（乙組）: mean 平均數 $= 47.5$kg; $\sigma = 1.5$ kg
(b) Group A（甲組）. $\because$ The standard deviation of group A is greater than that of group B. 甲組的標準差較 B 組大。
4. (a) Girls (女孩): $\sigma = 11.06$. Boys (男孩): $\sigma = 17.45$ (b) Girl's group 女孩
5. (a) Company A（甲公司）: mean 平均數 $= \$10000$; $\sigma = \$1206$
Company B（乙公司）: mean 平均數 $= \$9500$; $\sigma = \$1054$
(b) Company B（乙公司）. $\because$ The standard deviation of B is less than that of A.
乙公司的標準差比甲公司的標準差小。
6. (a) June 六月
(b) Jun (6 月): $\sigma_6 = 2.29$ h, Nov(11 月): $\sigma_{11} = 3.42$ h. Yes 是, $\because \sigma_6 < \sigma_{11}$.

10-6

1. (a) P (b) P
2. (a) C_2 (b) C_1

3. (a) I.


II.


III.


(b) II, III, I

10-7

1.

	mean 平均數	S.D. 標準差	range 分佈域	interquartile range 四分位數間距
(a)	10.6	1.48	13.5	4.2
(b)	61.4	28.5	46	20
(c)	11.2	10.9	27	22
	$11.2 + k$	10.9	27	22

2.

	mean 平均數	S.D. 標準差	range 分佈域	interquartile range 四分位數間距
(a)	$2a$	$2b$	$2c$	$2d$
(b)	$\frac{w}{5}$	$\frac{x}{5}$	$\frac{y}{5}$	$\frac{z}{5}$
(c)	37.4	25.0	69	48.5
	$37.4k$	$25k$	$69k$	$48.5k$

3. (a)

	Set 1 (第一組)		Set 2 (第二組)	
	mean 平均數	S.D. 標準差	mean 平均數	S.D. 標準差
original 原本的	9.5	2.69	8.63	6.26
the smallest removed 刪去最小的	10.4	1.18	9.43	6.30
the largest removed 刪去最大的	9.14	2.70	6.43	2.50

(b) Both of the means increase. 兩個平均數都增加。

(c) Both of the means decrease. 平均數皆減少。

(d) The standard deviation may increase or decrease, and there is no specific pattern.
標準差可能增加或減少，沒有特定的規律。

10-8

1. (a) 1.25 (b) –1.33 (c) 3 (d) 0.67 (e) 0.9
2. (a) 1.03 (b) 18.94 (c) 13.8 (d) 1.61
3. mean 平均數 = 61.6, $\sigma = 7.12$

Student 學生	A	B	C	D	E	F	G	H
Standard Score 標準分	–0.65	–0.09	–1.21	1.04	1.88	0.19	0.05	–1.21

10-9

1. $\frac{8.91-8.4}{0.45}=1.13$

2. $\frac{51-\mu}{9}=0.38$, $\mu=47.6$

3. (a) Liberal Studies 通識: –0.69
   Chinese History 中史: –1.01
   (b) Liberal Studies 通識

4. (a) Physics 物理: 1.69
   Biology 生物: 1.46
   (b) Physics 物理

5. (a) mean 平均數 $=\frac{79.5\times1+89.5\times5+99.5\times14+109.5\times1+119.5\times2}{30}=101.17$,
   $\sigma=8.96$
   (b) $\frac{103-101.17}{8.96}=0.20$

6. $\begin{cases}\frac{53-\mu}{\sigma}=-1.2.......(1)\\ \frac{71-\mu}{\sigma}=1.7..........(2)\end{cases}$; $\sigma=6.2$, $\mu=60.4$

10-10

1. (a) 34% (b) 81.5% (c) 0.15% (d) 2.5%
2. (a) 83.85% (b) 84% (c) 2.5% (d) 18.5%
3. (a) $\mu-2\sigma$ (b) $\mu+\sigma$ (c) $\mu+3\sigma$ (d) $\mu-\sigma$

10-11

1. (a) $47=55-8=\mu-\sigma$, $63=55+8=\mu+\sigma$. $\therefore$ % of students 學生的百分比 $=68\%$
   (b) $39=55-2\times8=\mu-2\sigma$. $\therefore$ % of students 學生的百分比 $=97.5\%$
   (c) $79=55+3\times8=\mu+3\sigma$. $\therefore$ % of students 學生的百分比 $=0.15\%$
   (d) $100\%-95\%=5\%$
2. (a) $500\times68\%=340$
   (b) $89=96-2\times3.5=\mu-2\sigma$, $106.5=96+3\times3.5=\mu+3\sigma$;
   $\therefore$ % of batteries 電池的百分比 $=500\times97.35\%=487$
   (c) $85.5=96-3\times3.5=\mu-3\sigma$; $\therefore$ no. of batteries 電池的數目 $=500\times0.15\%=1$
3. (a) $40=54-\sigma$, $\sigma=14$ (b) $54+2\times14=82$
   (c) minimum mark to get grade B（B 級的最低分數）$=54+14=68$.
   $\because 68<79<82$, $\therefore$ Mandy will get grade B. 曼儀將會得 B 級。

10-12

1. (a)

Marks 分數	Class mark 組中點	Frequency 頻數
1 – 4	2.5	**2**
5 – 8	**6.5**	5
9 – 12	**10.5**	**7**
13 – 16	**14.5**	**11**
17 –20	**18.5**	6
21 – 24	**22.5**	**4**
25 – 28	**26.5**	**3**

Mark less than 分數少於	Cumulative frequency 累積頻數
4.5	2
8.5	**7**
12.5	14
16.5	25
20.5	**31**
24.5	35
28.5	**38**

(b) 13 – 16

(c) mean 平均數 $= 14.5$, $\sigma = 6.29$

(d) (i) $\dfrac{21-14.5}{6.29} = 1.03$

(ii) $14.5 + 2 \times 1.03 \times 6.29 = 27.5$

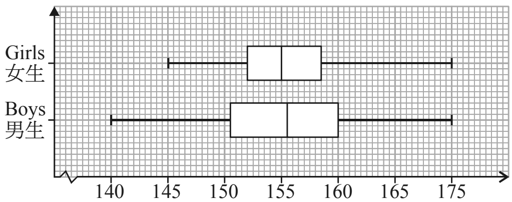
(e) standard score in History 歷史科的標準分 $= \dfrac{29.5-24}{6} = 0.92 < 1.03$

$\therefore$ Maggie performs better in the Mathematics test. 美琪數學科的表現較好。

2. (a) Boys 男生: 56; Girls 女生: 60

(b) (i) Boys 男生: median 中位數 $= 156$ cm, $Q_1 = 150.5$ cm, $Q_3 = 160$ cm

(ii) Girls 女生: median 中位數 $= 155$ cm, $Q_1 = 152$ cm, $Q_3 = 158.5$ cm



(iii) Since the range and the interquartile range of the data of boys are larger than that of girls. 因男生的分佈域及四分位數間距較女生大。

$\therefore$ the height distribution of the boys is more disperse.
男生高度的離差較大。

(c) (i) $5 + 12 = 17$

(ii) Total no. of boys and girls shorter than 身高 163 cm 以下的男生及女生

$= 46 + 56 = 102$. The required 所求 % $= \dfrac{116-102}{116} \times 100\% = 12.1\%$

3. (a) mean 平均數: decreases 減少, median 中位數: unchanged 不變,
standard deviation 標準差: decrease 減少

(b)

	mean 平均數	σ
Jason 志森	528 s	56.8 s
Simon 世民	535 s	61.7 s

(c)

	median 中位數	I.Q.R 四分位間距
Jason 志森	$\dfrac{533+555}{2} = 544$ s	572 – 487 = 85 s
Simon 世民	$\dfrac{521+537}{2} = 529$ s	567 – 488 = 79 s

(d) (i) Jason's data. 志森的數據。

(ii) Simon's data. 世民的數據

(iii) Because there is an extreme data (684 s) in Simon's records.
因為世民的紀錄中有一個極端的數據(684 s)。

10-13

1. x: required mean score 所需平均分. $\dfrac{44 \times 3 + x \times 2}{3+2} = 60$, $x = 84$

2. $\dfrac{(a+11)+(b-4)+(c+2)}{3} = 7$, $a+b+c = 12$. $\dfrac{(a+9)+(b-5)+(c+6)}{3} = \dfrac{12+10}{3} = \dfrac{22}{3}$

3. $x^2 - 5 < x^2 - 3 < x^2 < x^2 + 1 < x^2 + 2$, median 中位數 $= x^2$, range 分佈域 $= 7$

4. (a) $\frac{64m+72n}{m+n}=69$, $3n=5m$, $m:n=3:5$ (b) $=40\times\frac{5}{3+5}=25$

5. 3σ

6. $=\sqrt{\frac{(x-\frac{x+y}{2})^2+(y-\frac{x+y}{2})^2}{2}}=\sqrt{\frac{(\frac{x-y}{2})^2+(\frac{y-x}{2})^2}{2}}=\sqrt{\frac{(y-x)^2}{4}}=\frac{y-x}{2}$ $(\because y-x>0)$

7. $15>9$ and 和 $n-4<n-3<n-1<n+18$, $\therefore$ $n-1=9$, $n=10$;
$\therefore$ mean 平均$=13$; s.d.標準差$=8.12$

8. $\frac{48-60}{\sigma}=-0.6$, $\sigma=20$

9. $\frac{78-\bar{x}}{7}=1.08$, $\bar{x}=70.44$

10. $\frac{60-65}{\sigma}=-0.84$, $\sigma=5.952381$; $\frac{88-65}{5.952381}=3.864$

11. (a) 11
(b) $4+10+6<n+5$, $n>15$, $\because$ n is integer (n 是整數), $\therefore$ $n=16, 17, 18, \ldots$..
(c) $\frac{0\times4+1\times10+2\times6+3n+4\times5}{4+10+6+n+5}=2$, $n=8$; $\therefore$ $4+10+6+8+5=33$

12. (a) 40; $\frac{127}{3}$; 41.5; 7
(b) (i) least 最小$=\frac{40+40}{2}=40$; greatest 最大$=\frac{44+47}{2}=45.5$
(ii) 38, 39, 40, 40, 41, 42, 44, 45, 47, 50. $Q_3-Q_1=45-40=5$

13. (a) No. Both intervals contain 25% of the students.
不是，兩個間距均包含 25%的學生。
(b) $58.7-x=11.2$, $x=47.5$; $y=50$